MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

# LINCOLN LABORATORY ANALYSES OF PARABOLOIDAL SHELLS (LLAPS) USER'S MANUAL

LINCOLN MANUAL 60

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

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Enclosed herewith is Sec. IV to be added to your copy of the manual. Please replace pages ii and 49 of your manual with the enclosed, revised pages.

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## TABLE OF CONTENTS



Rev. 17 November 1965

# IV. SHALLOW SHELL ANALYSIS OF GRAVITY LOAD

## A. SHALLOW SHELL SOLUTION

A paraboloidal shell of revolution is considered shallow if

$$\left(\frac{dy_3}{dr}\right)^2 = \gamma^2 << 1 \tag{IV-1}$$

so that

$$1 + \gamma^2 \approx 1 \quad . \tag{IV-2}$$

If terms involving transverse shears are neglected in the tangential equilibrium equations and terms involving tangential displacements are neglected in the expression for the rotations, a consistent application of (IV-2) leads to the following two simultaneous linear partial differential equations

$$A\nabla^2\nabla^2\Phi = -\frac{1}{2f}\nabla^2 w + A(1-\nu)\nabla^2 P \tag{IV-3}$$

$$D\nabla^2\nabla^2 w = \frac{1}{2f}\nabla^2\Phi - \frac{1}{f}P + p_3 \tag{IV-4}$$

where

$$\nabla^2(\,) = \frac{\partial^2(\,)}{\partial r^2} + \frac{1}{r}\frac{\partial(\,)}{\partial r} + \frac{1}{r^2}\frac{\partial^2(\,)}{\partial\Theta^2}$$

and $P$ is a load potential such that

$$\frac{\partial P}{\partial r} = p_r \tag{IV-5}$$

$$\frac{1}{r}\frac{\partial P}{\partial\Theta} = p_\Theta \quad . \tag{IV-6}$$

For gravity load

$$P = 2f\rho h\left(\gamma \sin\Theta \sin\psi - \frac{\gamma^2}{2}\cos\psi\right) \tag{IV-7}$$

$\Phi$ is a stress function from which the membrane stress resultants are derivable

$$N_r = \left[\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\Theta^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r}\right] - P \tag{IV-8}$$

$$N_\Theta = \left[\frac{\partial^2\Phi}{\partial r^2}\right] - P \tag{IV-9}$$

$$N_{r\Theta} = \left[\frac{1}{r^2}\frac{\partial\Phi}{\partial\Theta} - \frac{1}{r}\frac{\partial^2\Phi}{\partial r\partial\Theta}\right] \quad . \tag{IV-10}$$

The transverse shears and moment resultants are related to the transverse deflection $w$ by

$$M_r = -D\left[\frac{\partial^2 w}{\partial r^2} + \nu\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\Theta^2}\right)\right] \tag{IV-11}$$

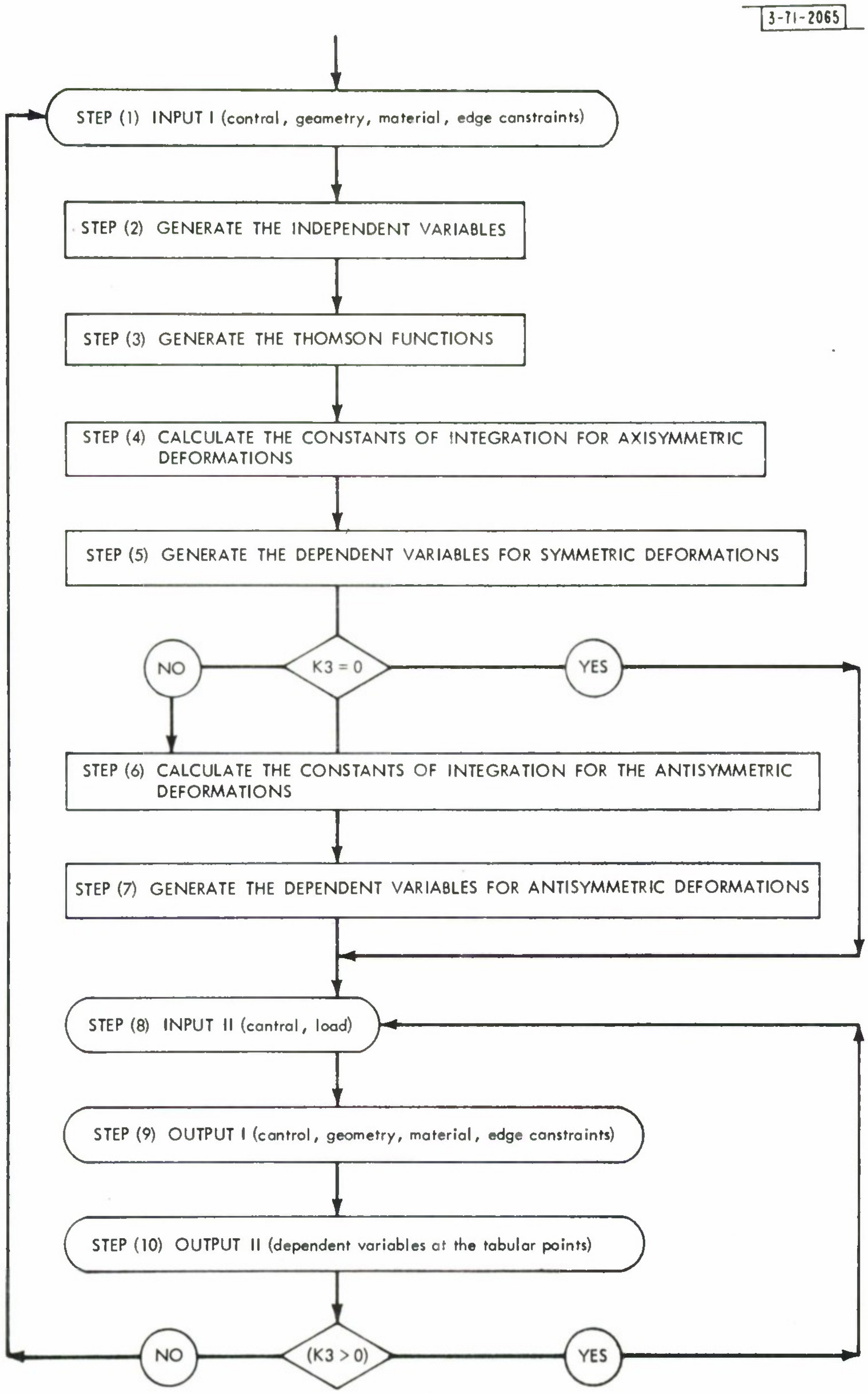



Fig. 13. Master flow chart.

$$M_\Theta = -D\left[\nu \frac{\partial^2 w}{\partial r^2} + \left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \Theta^2}\right)\right] \qquad \text{(IV-12)}$$

$$M_{r\Theta} = -D(1-\nu)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial \Theta}\right) \qquad \text{(IV-13)}$$

$$Q_r = -D\frac{\partial}{\partial r}(\nabla^2 w) \qquad \text{(IV-14)}$$

$$Q_\Theta = -D\frac{1}{r}\frac{\partial}{\partial \Theta}(\nabla^2 w) \quad . \qquad \text{(IV-15)}$$

To be consistent with the shallow shell approximation, the rotations must be approximated as follows [see Eqs. (II-28) and (II-29)]:

$$\omega_r = -\frac{\partial w}{\partial r} \qquad \text{(IV-16)}$$

$$\omega_\Theta = -\frac{1}{r}\frac{\partial w}{\partial \Theta} \quad . \qquad \text{(IV-17)}$$

Having $\Phi$ and w, the two tangential middle-surface-displacement components u and v can be obtained by integrating the strain-displacement relations for a shallow shell corresponding to (II-25), (II-26), and (II-27).

The solution to (IV-3) and (IV-4) involves Thomson functions of order zero and their derivatives. While these functions have been tabulated in various sources (most extensively in Ref. 6), none of these tabulations is complete for our purpose. For any particular shell, we may need values of these functions not tabulated in the given reference. Subroutines have been written in double precision arithmetic to generate the values of these functions for any argument (within the limitations of the IBM 7094). Because of the double precision arithmetic and the nature of the computations involved, these subroutines are extremely time consuming. On the other hand, the nature of the Thomson functions is such that they contribute significantly to the complete solution only in the neighborhood of the edges of the shell. In these regions, the magnitude of these functions varies rapidly over a short interval of the independent variable. To get sufficient information about the shell behavior, it is necessary to have smaller increments between the $\gamma$-tabular points in these regions. In contrast, the solution in the interior varies slowly over the span of the shell, so that we do not need the same density of $\gamma$-tabular points. A waste of computer time can and will be avoided by changing the increment of the $\gamma$-tabular points, once we move away from the edge.

## B. DIGITAL COMPUTER PROGRAM

The general scheme of the program for a shallow shell analysis is outlined in the master flow chart (Fig. 13). When it fails to locate additional input at step (1) or step (8), the program exits automatically. The program listing is presented in Table V.

TABLE V

PROGRAM LISTING

```
*      SHEA,ELAINE  SHALLOW SHELL WITH GRAVITY LOAD
*      XEQ
C      SHEA,ELAINE  SHALLOW SHELL WITH GRAVITY LOAD
C
C
C
       DIMENSION  R(50), Y(50),BBER(50),BBEI(50),AAKER(50),AAKEI(50),X(2)
     1,AAKERP(50),AAKEIP(50),BBRP(50),BBIP(50),SBER(50),SBEI(50),SKER(50
     2),SKEI(50),TBER(50),TBEI(50),TKER(50),TKEI(50),WW(50),W3(50),
     3 W(50,10),WP(50,10),V(50,10),U(50,10),CMR(50,10),CMT(50,10),
     4CMRT(50,10),CNR(50,10),CNT(50,10),CNRT(50,10),QR(50,10),QT(50,10),
     5VV(50),UU(50),Z3(8),DMR(50),DMT(50),DMRT(50),DNR(50),DNT(50),DNRT(
     650),QQR(50),QQT(50),P(50),W1(50),Q(50),EMR(50),EMT(50),ENR(50),ENT
     7(50),Q1(50),A(8,8),S(8),AA(4,4),SS(4),ZZ(4),B(6,6),BB(6),B9(6),D(3
     8,3),DD(3),D3(3),RG(50)
       DIMENSION P2(50),W2(50),FMR(50),FNR(50),Q2(50),FMT(50),FNT(50),Q3
     1(50),W4(50),W5(50),GMR(50),GMT(50),GMRT(50),QRR(50),QTT(50),GNR(50
     2),GNT(50),GNRT(50),U2(50),V2(50),WT(50),WFLEXT(50),WPT(50),UT(50)
       DIMENSION  THETA(20),THETR(20),FMT1(12),FMT2(12)
       DIMENSION  PWB(50),WA5(50),WFLEX(50,10),PWB2(50),WA54(50)
       COMMON  W,WP,V,U,CMR,CMT,CMRT,CNR,CNRT,CNT
       EQUIVALENCE  (BBER,P2),(BBEI,W2),(AAKER,FMR),(AAKEI,FNR),(AAKERP,
     1Q2),(AAKEIP,FMT),(BBRP,FNT),(BBIP,Q3),(SBER,W4),(SBEI,W5),(SKER,
     2GMR),(SKEI,GMT),(TBER,GMRT),(TBEI,QRR),(TKER,QTT),(TKEI,GNR)
C
C - - (1) INPUT I (CONTROL,GEOMETRY,MATERIAL,EDGE CONSTRAINT)
C
   98 READ INPUT TAPE 2,101,M,MU,ML,MC
      READ INPUT TAPE 2,101,N,NORM,K1,K2,K3
      READ INPUT TAPE 2,102,RHO,E,PRT
      READ INPUT TAPE 2,103,R1,R2,R3,R4
      READ INPUT TAPE 2,103,THETA3,THETA4,F,H
      IF(N)182,182,181
  181 READ INPUT TAPE 2,205,FMT1,FMT2
  182 WRITE OUTPUT TAPE 3,10
C
C - - (2) GENERATE THE INDEPENDENT VARIABLES
C
      F2=2.*F
      F1=1./F2
      C=SQRTF(12.*(1.-PRT**2))
      C=SQRTF(C)
      C=C/SQRTF(F2*H)
      PI=3.1415926535
      RAD=57.2957795
      R5=0.
      R6=0.
      IF(MC-1)76,77,77
   77 R5=F2*H
      R5=2.*SQRTF(R5)
      EM=MU
      DRB=R5/EM
      R(M)=R4
      DO 1 I=1,MU
      J=M-I
    1 R(J)=R(J+1)-DRB
      IF(ML-1)76,76,78
   78 EM=ML
```

TABLE V (Continued)

```
      DRB=R5/EM
      R(1)=R3
      DO 931 I=1,ML
  931 R(I+1)=R(I)+DRB
      R6=R5
   76 MM=M-ML-MU
      M1=MM-1
      EM1=M1
      DRB=R4-R3-R5-R6
      DRB=DRB/EM1
      MM1=ML+1
      R(MM1)=R3+R6
      J=MM1
      DO 932 I=1,M1
      J=J+1
  932 R(J)=R(J-1)+DRB
      IF(NORM)680,680,682
  680 DO 681 I=1,M
      RG(I)=R(I)/F2
  681 Y(I)=R(I)/F2
      GO TO 684
  682 DO 683 I=1,M
      RG(I)=R(I)/F2
  683 Y(I)=R(I)
  684 G=E*H**3/(12.*(1.-PRT**2))
      DRB=DRB/F2
      PM=PRT-1.
      PP=PRT+1.
      P1=1.-PRT
      T=H/SQRTF(12.*(1.-PRT**2))
      IF(N)84,84,46
   46 THETA3=THETA3*PI
      THETA4=THETA4*PI
      N1=N-1
      AN1=N1
      THETA5=THETA4-THETA3
      DELTT=THETA5/AN1
      THETA(1)=THETA3
      DO 47 J=1,N1
      J1=J+1
   47 THETA(J1)=THETA(J)+DELTT
C
C - - (3) GENERATE THE THOMSON FUNCTIONS
C
   84 DO 5 I=1,M
      X=C*R(I)
      X1=1./X
      IF(X-6.0)2,2,3
    2 LONG=10
      GO TO 4
    3 LONG=16
    4 BBER(I)=BER(X,LONG)
      BBEI(I)=BEI(X,LONG)
      BBRP(I)=BRP(X,LONG)
      BBIP(I)=BIP(X,LONG)
      AAKER(I)=AKER(X,LONG)
      AAKEI(I)=AKEI(X,LONG)
      AAKERP(I)=AKERP(X,LONG)
```

TABLE V (Continued)

```
      AAKEIP(I)=AKEIP(X,LONG)
      SBER(I)=-BBEI(I)-X1*BBRP(I)
      SBEI(I)=BBER(I)-X1*BBIP(I)
      SKER(I)=-AAKEI(I)-X1*AAKERP(I)
      SKEI(I)=AAKER(I)-X1*AAKEIP(I)
      TBER(I)=BBEI(I)/X-BBIP(I)+2.*BBRP(I)/X**2
      TBEI(I)=BBRP(I)-BBER(I)/X+2.*BBIP(I)/X**2
      TKER(I)=AAKEI(I)/X-AAKEIP(I)+2.*AAKERP(I)/X**2
    5 TKEI(I)=AAKERP (I)-AAKER(I)/X+2.*AAKEIP(I)/X**2
C
C - - (4) CALCULATE THE CONSTANTS OF INTEGRATION FOR THE AXI-SYMMETRIC
C       DEFORMATIONS
C
      IF(NORM)32,32,31
   31 CNO=1.
      CNO1=1.
      CNO2=1.
      CNO3=1.
      CNO8=1.
      GO TO 202
   32 CNO=F2*RHO*H
      CNO1=E/(F2**2*RHO)
      CNO2=F2*RHO*H**2
      CNO3=RHO*H*SQRTF(F2*H)
      CNO8=RHO*F2**2/(E*SQRTF(F2*H))
  202 Z=R3
      X=C*Z
      X1=1./X
      X2=C/Z
      L=1
      MM=1
      IF(K1-2)29,28,85
   85 IF(K1-3)28,28,34
   26 Z=R4
      X=C*Z
      X1=1./X
      X2=C/Z
      MM=M
      IF(K2-2)29,28,28
C     W(R)=0.
   28 B(L,1)=BBER(MM)
      B(L,2)=BBEI(MM)
      B(L,3)=AAKER(MM)
      B(L,4)=AAKEI(MM)
      B(L,5)=1.
      B(L,6)=0.
      BB(L)=RHO*P1*Z**2*.5/E
      L=L+1
C     U(R)=0.
      B(L,1)=X1*PP*BBIP(MM)
      B(L,2)=-X1*PP*BBRP(MM)
      B(L,3)=X1*PP*AAKEIP(MM)
      B(L,4)=-X1*PP*AAKERP(MM)
      B(L,5)=1.
      B(L,6)=-PP*X1**2
      BB(L)=2.*F**2*RHO*PM/E
      L=L+1
      IF(L-3)203,203,56
```

TABLE V (Continued)

```
  203 IF(K1-3)30,29,34
   56 IF(K2-3)30,29,34
C      WP(R)=0.
   30 B(L,1)=C*BBRP(MM)
      B(L,2)=C*BBIP(MM)
      B(L,3)=C*AAKERP(MM)
      B(L,4)=C*AAKEIP(MM)
      B(L,5)=0.
      B(L,6)=0.
      BB(L)=RHO*P1*Z/E
      L=L+1
      IF(L-4)26,26,36
C      MR(R)=0.
   29 B(L,1)=-(C**2)*BBEI(MM)+PM*X2*BBRP(MM)
      B(L,2)=C**2*BBER(MM)+PM*X2*BBIP(MM)
      B(L,3)=-(C**2)*AAKEI(MM)+PM*X2*AAKERP(MM)
      B(L,4)=C**2*AAKER(MM)+PM*X2*AAKEIP(MM)
      B(L,5)=0.
      B(L,6)=0.
      BB(L)=RHO*H**3/(12.*G)
      L=L+1
      IF(L-4)35,26,73
   73 IF(L-5)35,35,36
C      NR(R)=0.
   35 B(L,1)=-X2*BBIP(MM)
      B(L,2)=X2*BBRP(MM)
      B(L,3)=-X2*AAKEIP(MM)
      B(L,4)=X2*AAKERP(MM)
      B(L,5)=0.
      B(L,6)=1./Z**2
      BB(L)=-F*RHO/(E*T)
      L=L+1
C      QR(R)=0.
      B(L,1)=-C**3*BBIP(MM)
      B(L,2)=C**3*BBRP(MM)
      B(L,3)=-C**3*AAKEIP(MM)
      B(L,4)=C**3*AAKERP(MM)
      B(L,5)=0.
      B(L,6)=0.
      BB(L)=0.
      L=L+1
      IF(L-4)26,26,36
   36 DET=1.
      MATR=XSIMEQF(6,6,1,B,BB,DET,B9)
      GO TO (6,7,8),MATR
    6 B1=B(1,1)
      B2=B(2,1)
      B3=B(3,1)
      B4=B(4,1)
      B5=B(5,1)
      B6=B(6,1)
      GO TO 42
   34 X2=1./(C*R2)
      X3=C/R2
C      W(R2)=0.
      D(1,1)=BBER(M)
      D(1,2)=BBEI(M)
      D(1,3)=1.
```

```
      DD(1)=RHO*P1*R2**2*.5/E
C     U(R2)=0.
      D(2,1)=X2*PP*BBIP(M)
      D(2,2)=-X2*PP*BBRP(M)
      D(2,3)=1.
      DD(2)=2.*F**2*RHO*PM/E
      IF(K2-2)37,37,38
   37 D(3,1)=C*BBRP(M)
      D(3,2)=C*BBIP(M)
      D(3,3)=0.
      DD(3)=RHO*P1*R2/E
      GO TO 43
C     MR(R2)=0.
   38 D(3,1)=-C**2*BBEI(M)+PM*X3*BBRP(M)
      D(3,2)=C**2*BBER(M)+PM*X3*BBIP(M)
      D(3,3)=0.
      DD(3)=RHO*H**3/(12.*G)
   43 DET=1.
      MATR=XSIMEQF(3,3,1,D,DD,DET,D3)
      GO TO (44,7,8),MATR
   44 B1=D(1,1)
      B2=D(2,1)
      B3=0.
      B4=0.
      B5=D(3,1)
      B6=0.
C
C - - (5) GENERATE THE DEPENDENT VARIABLES FOR THE AXI-SYMMETRIC DEFORMA
C     TIONS
C
   42 DO 9 I=1,M
      X=C*R(I)
      X1=1./X
      P(I)=B1*BBER(I)+B2*BBEI(I)+B3*AAKER(I)+B4*AAKEI(I)+B5-RHO*P1
     1*R(I)**2*.5/E
      PWB(I)=(P(I)-B5)*CNO1
      P(I)=P(I)*CNO1
      W1(I)=B1*C*BBRP(I)+B2*C*BBIP(I)+B3*C*AAKERP(I)+B4*C*AAKEIP(I)-RHO*
     1P1*R(I)/E
      W1(I)=W1(I)/CNO8
      CMR1=B1*(-C**2*BBEI(I)+PM*C/R(I)*BBRP(I))+B2*(C**2*BBER(I)+PM*C/R(
     1I)*BBIP(I) )
      EMR(I)=-G*(CMR1+B3*(-C**2*AAKEI(I)+PM*C/R(I)*AAKERP(I))+B4*(C**2*A
     1AKER(I)+PM*C/R(I)*AAKEIP(I)))+RHO*H**3/12.
      EMR(I)=EMR(I)/CNO2
      ENR(I)=E*T*H*(B2*C/R(I)*BBRP(I)-B1*C/R(I)*BBIP(I)-B3*C/R(I)*AAKEIP
     1(I)+B4*C/R(I)*AAKERP(I)+B6/R(I)**2)+F*RHO*H
      ENR(I)=ENR(I)/CNO
      Q1(I)=-G*(-B1*C**3*BBIP(I)+B2*C**3*BBRP(I)-B3*C**3*AAKEIP(I)+B4*C*
     1*3*AAKERP(I))
      Q1(I)=Q1(I)/CNO3
      CMT1=B1*(-PRT    *C**2*BBEI(I)+P1*C/R(I)*BBRP(I))+B2*(PRT    *C**2*B
     1BER(I)+P1*C/R(I)*BBIP(I))
      EMT(I)=-G*(CMT1+B3*(-PRT    *C**2*AAKEI(I)+P1*C/R(I)*AAKERP(I))+B4*
     1(PRT    *C**2*AAKER(I)+P1*C/R(I)*AAKEIP(I)))+RHO*H**3/12.
      EMT(I)=EMT(I)/CNO2
      ENT1=B1*(-BBER(I)+X1*BBIP(I))-B2*(BBEI(I)+X1*BBRP(I))+B3*(-AAKER(I
     1)+X1*AAKEIP(I))
```

TABLE V (Continued)

```
      ENT(I)=E*H*F1*(ENT1-B4*(AAKEI(I)+X1*AAKERP(I))-B6/X**2)+F*RHO*H
      ENT(I)=ENT(I)/CNO
      U1=B1*X1*BBIP(I)-B2*X1*BBRP(I)+B3*X1*AAKEIP(I)-B4*X1*AAKERP(I)+B5/
     1PP-B6*X1**2
      Q(I)=R(I)*F1*PP*U1+F*RHO*P1*R(I)/E
      Q(I)=Q(I)*CNO1
    9 CONTINUE
      IF(K3)402,402,45
C
C - - (6) CALCULATE THE CONSTANTS OF INTEGRATION FOR THE ANTI-SYMMETRIC
C     DEFORMATIONS
C
   45 Z=R3
      X=C*Z
      X1=1./X
      L=1
      MM=1
      IF(K1-2)54,53,87
   48 Z=R4
      X=C*Z
      X1=1./X
      MM=M
      IF(K2-2)54,53,53
   87 IF(K1-3)53,53,60
C     W(R1)=0.
   53 A(L,1)=BBRP(MM)
      A(L,2)=BBIP(MM)
      A(L,3)=AAKERP(MM)
      A(L,4)=AAKEIP(MM)
      A(L,5)=X1
      A(L,6)=X
      A(L,7)=0.
      A(L,8)=0.
      S(L)=0.
      L=L+1
C     U(R)=0.
      A(L,1)=PP*SBEI(MM)
      A(L,2)=-PP*SBER(MM)
      A(L,3)=PP* SKEI(MM)
      A(L,4)=-PP* SKER(MM)
      A(L,5)=2.*LOGF(X)
      A(L,6)=X**2*.5
      A(L,7)=PP*X1**2
      A(L,8)=1.
      S(L)=.25*X*Z*F*RHO*(1.+5.*PRT)/E
      L=L+1
C     V(R)=0.
      A(L,1)=PP*X1*BBIP(MM)
      A(L,2)=-PP*X1*BBRP(MM)
      A(L,3)=PP*X1*AAKEIP(MM)
      A(L,4)=-PP*X1*AAKERP(MM)
      A(L,5)=2.*LOGF(X)
      A(L,6)=-X**2*.5
      A(L,7)=-PP*X1**2
      A(L,8)=1.
      S(L)=.25*X*Z*F*RHO*(11.+7.*PRT)/E
      L=L+1
      IF(L-4)57,57,58
```

TABLE V (Continued)

```
   57 IF(K1-3)55,54,60
   58 IF(K2-3)55,54,60
C     WP(R)=0.
   55 A(L,1)=SBER(MM)
      A(L,2)=SBEI(MM)
      A(L,3)=SKER(MM)
      A(L,4)=SKEI(MM)
      A(L,5)=-X1**2
      A(L,6)=1.
      A(L,7)=0.
      A(L,8)=0.
      S(L)=0.
      L=L+1
      IF(L-5)48,48,61
C     MR(R)=0.
   54 A(L,1)=TBER(MM)+PRT    *X1*(SBER(MM)-X1*BBRP(MM))
      A(L,2)=TBEI(MM)+PRT    *X1*(SBEI(MM)-X1*BBIP(MM))
      A(L,3)=TKER(MM)+PRT    *X1*(SKER(MM)-X1*AAKERP(MM))
      A(L,4)=TKEI(MM)+PRT    *X1*(SKEI(MM)-X1*AAKEIP(MM))
      A(L,5)=P1*X1**3
      A(L,6)=0.
      A(L,7)=0.
      A(L,8)=0.
      S(L)=0.
      L=L+1
      IF(L-5)62,48,63
   63 IF(L-6)62,62,61
C     NR(R)=0.
   62 A(L,1)=BBER(MM)-2.*X1*BBIP(MM)
      A(L,2)=BBEI(MM)+2.*X1*BBRP(MM)
      A(L,3)=AAKER(MM)-2.*X1*AAKEIP(MM)
      A(L,4)=AAKEI(MM)+2.*X1*AAKERP(MM)
      A(L,5)=-1./PP
      A(L,6)=0.
      A(L,7)=2.*X1**2
      A(L,8)=0.
      S(L)=-X*Z*RHO*F*.5/E
      L=L+1
C     QR(R)+1./ALPHA2*DMRT/DTHETA=0.
      A(L,1)=G*C*SBEI(MM)-(G*P1*X1/Z)*(BBEI(MM)+2.*X1*BBRP(M))
      A(L,2)=-G*C*SBER(MM)+(G*P1*X1/Z)*(BBER(MM)-2.*X1*BBIP(M))
      A(L,3)=G*C*SKEI(MM)-(G*P1*X1/Z)*(AAKEI(MM)+2.*X1*AAKERP(MM))
      A(L,4)=-G*C*SKEI(MM)+(G*P1*X1/Z)*(AAKER(MM)-2.*X1*AAKEIP(MM))
      A(L,5)=-PM*G*C**2*F1*X1/(SQRTF(1.+(Z*F1)**2))**3*2.*X1**2
      A(L,6)=0.
      A(L,7)=0.
      A(L,8)=0.
      S(L)=0.
      L=L+1
C     NRT(R)=0.
      A(L,1)=BBER(MM)-2.*X1*BBIP(MM)
      A(L,2)=BBEI(MM)+2.*X1*BBRP(MM)
      A(L,3)=AAKER(MM)-2.*X1*AAKEIP(MM)
      A(L,4)=AAKEI(MM)+2.*X1*AAKERP(MM)
      A(L,5)=1./PP
      A(L,6)=0.
      A(L,7)=2.*X1**2
      A(L,8)=0.
```

TABLE V (Continued)

```
      S(L)=1.5*RHO*F*X*Z/E
      L=L+1
      IF(L-5)48,48,61
   61 DET=1.
      MATR=XSIMEQF(8,8,1,A,S,DET,Z3)
      GO TO (206,7,8),MATR
  206 A1=A(1,1)
      A2=A(2,1)
      A3=A(3,1)
      A4=A(4,1)
      A5=A(5,1)
      A6=A(6,1)
      A7=A(7,1)
      A8=A(8,1)
      GO TO 68
   60 IF(K2-2)64,64,65
C     WP(R2)=0.
   64 AA(1,1)=SBER(M)
      AA(1,2)=SBEI(M)
      AA(1,3)=1.
      AA(1,4)=0.
      SS(1)=0.
      GO TO 66
C     MR(R2)=0.
   65 AA(1,1)=TBER(M)+PRT    /(C*R2)*(SBER(M)-BBRP(M)/(C*R2))
      AA(1,2)=TBEI(M)+PRT    /(C*R2)*(SBEI(M)-BBIP(M)/(C*R2))
      AA(1,3)=0.
      AA(1,4)=0.
      SS(1)=0.
C     W(R2)=0.
   66 AA(2,1)=BBRP(M)
      AA(2,2)=BBIP(M)
      AA(2,3)=C*R2
      AA(2,4)=0.
      SS(2)=0.
C     V(R2)=0.
      AA(3,1)=PP*BBIP(M)/(C*R2)
      AA(3,2)=-PP*BBRP(M)/(C*R2)
      AA(3,3)=-.5*(C*R2)**2
      AA(3,4)=1.
      SS(3)=.25*C*R2**2*F*RHO*(11.+7.*PRT)/E
C     U(R2)=0.
      AA(4,1)=PP*SBEI(M)
      AA(4,2)=-PP*SBER(M)
      AA(4,3)=.5*(C*R2)**2
      AA(4,4)=1.
      SS(4)=.25*C*R2**2*F*RHO*(1.+5.*PRT)/E
      DET=1.
      MATR=XSIMEQF(4,4,1,AA,SS,DET,ZZ)
      GO TO (67,7,8),MATR
   67 A1=AA(1,1)
      A2=AA(2,1)
      A3=0.
      A4=0.
      A5=0.
      A6=AA(3,1)
      A7=0.
      A8=AA(4,1)
```

```
      GO TO 68
    7 WRITE OUTPUT TAPE 3,120
      GO TO 99
    8 WRITE OUTPUT TAPE 3,121
      GO TO 99
C
C - - (7) GENERATE THE DEPENDENT VARIABLES FOR THE ANTI-SYMMETRIC DEFORM
C     ATIONS
C
   68 DO 71 I=1,M
      X=R(I)*C
      X1=1./X
      WW(I)=A1*BBRP(I)+A2*BBIP(I)+A3*AAKERP(I)+A4*AAKEIP(I)+A5*X1+A6*X
      WA5(I)=(WW(I)-A6*X)*CNO1
      WW(I)=WW(I)*CNO1
      W3(I)=C*(A1*SBER(I)+A2*SBEI(I)+A3*SKER(I)+A4*SKEI(I)-A5*X1**2+A6)
      W3(I)=W3(I)/CNO8
      DMR1=A1*(TBER(I)+PRT    *X1*(SBER(I)-X1*BBRP(I)))+A2*(TBEI(I)+PRT
     1  *X1*(SBEI(I)-X1*BBIP(I)))+A3*(TKER(I)+PRT    *X1*(SKER(I)-X1*AAK
     2ERP(I)))
      DMR(I)=-G*C**2*(DMR1+A4*(TKEI(I)+PRT    *X1*(SKEI(I)-X1*AAKEIP(I))
     1)+A5*(P1*X1**3))
      DMR(I)=DMR(I)/CNO2
      DMT1=A1*(PRT    *TBER(I)+X1*(SBER(I)-X1*BBRP(I)))+A2*(PRT    *TBEI
     1(I)+X1*(SBEI(I)-X1*BBIP(I)))+A3*(PRT    *TKER(I)+X1*(SKER(I)    -X1*
     2AAKERP(I)))
      DMT(I)=-G*C**2*(DMT1+A4*(PRT    *TKEI(I)+X1*(SKEI(I)-X1*AAKEIP(I)))
     1-A5*(P1*X1**3))
      DMT(I)=DMT(I)/CNO2
      DMRT1=-A1*(BBEI(I)+2.*X1*BBRP(I))+A2*(BBER(I)-2.*X1*BBIP(I))-A3*(
     1AAKEI(I)+2.*X1*AAKERP(I))
      DMRT(I)=PM*G*C**2*X1*(DMRT1+A4*(AAKER(I)-2.*X1*AAKEIP(I))-A5*(2.*
     1X1**2))
      DMRT(I)=DMRT(I)/CNO2
      QQR(I)=-G*C**3*(-A1*SBEI(I)+A2*SBER(I)-A3*SKEI(I)+A4*SKER(I))
      QQR(I)=QQR(I)/CNO3
      QQT(I)=-G*C**3*X1*(-A1*BBIP(I)+A2*BBRP(I)-A3*AAKEIP(I)+A4*AAKERP
     1(I))
      QQT(I)=QQT(I)/CNO3
      DNR1=A1*(BBER(I)-2.*X1*BBIP(I))+A2*(BBEI(I)+2.*X1*BBRP(I))+A3*(AA
     1KER(I)-2.*X1*AAKEIP(I))+A4*(AAKEI(I)+2.*X1*AAKERP(I))
      DNR(I)=-E*H*F1*X1*(DNR1-A5/PP+A7*(2.*X1**2))-RHO*H*R(I)/4.
      DNR(I)=DNR(I)/CNO
      DNT1=-A1*TBEI(I)+A2*TBER(I)-A3*TKEI(I)+A4*TKER(I)-A5/PP*X1+A7*(2.
     1*X1**3)
      DNT(I)=E*H*F1*DNT1+5.*RHO*H*R(I)/4.
      DNT(I)=DNT(I)/CNO
      DNRT(I)=E*H*F1*X1*(DNR1+A5/PP+A7*(2.*X1**2))-3.*RHO*H*R(I)/4.
      DNRT(I)=DNRT(I)/CNO
      U1=-PP*(-A1*SBEI(I)+A2*SBER(I)-A3*SKEI(I)+A4*SKER(I))+A5*(2.*LOGF
     1(X))+A6*X**2*.5+A7*(PP*X1**2)+A8
      UU(I)=F1/C*U1-RHO*(1.+5.*PRT)*R(I)**2/(8.*E)
      UU(I)=UU(I)*CNO1
      V1=-PP*X1*(-A1*BBIP(I)+A2*BBRP(I)-A3*AAKEIP(I)+A4*AAKERP(I))+A5*(
     12.*LOGF(X))-A6*(.5*X**2)-A7*(PP*X1**2)+A8
      VV(I)=F1/C*V1-RHO*(11.+7.*PRT)*R(I)**2/(8.*E)
   71 VV(I)=VV(I)*CNO1
C
```

TABLE V (Continued)

```
C - - (8) INPUT II (CONTROL,LOAD)
C
  402 READ INPUT TAPE 2,404,PSI,MGOTO
C
C - - (9) OUTPUT I (CONTROL,GEOMETRY,MATERIAL,EDGE CONSTRAINTS)
C
      K3=K3-1
      IF(NORM)184,184,183
  183 WRITE OUTPUT TAPE 3,118
      GO TO 185
  184 WRITE OUTPUT TAPE 3,119
  185 IF(K1-2)13,14,15
   13 WRITE OUTPUT TAPE 3,129
      GO TO 16
   14 WRITE OUTPUT TAPE 3,130
      GO TO 16
   15 IF(K1-4)17,22,22
   17 WRITE OUTPUT TAPE 3,131
      GO TO 16
   22 WRITE OUTPUT TAPE 3,136
   16 IF(K2-2)18,19,21
   18 WRITE OUTPUT TAPE 3,134
      GO TO 33
   19 WRITE OUTPUT TAPE 3,132
      GO TO 33
   21 WRITE OUTPUT TAPE 3,133
   33 WRITE OUTPUT TAPE 3,128
      WRITE OUTPUT TAPE 3,122,RHO,E,H,F,PRT
      WRITE OUTPUT TAPE 3,11
      TPSI=2.*PSI-1.
      PSIDEG=PSI*RAD*PI
      PSI=PSI*PI
      T3DEG=THETA3*RAD
      T4DEG=THETA4*RAD
      AR=3./(SQRTF(1.+(R4/F2)**2)**3-SQRTF(1.+(R3/F2)**2)**3)
      FM1=0.
      FM2=0.
      FM4=0.
      RM1=0.
      RM2=0.
      RM4=0.
      DO 3000 I=1,M
      RM=(P(I)*COSF(PSI))**2+(WW(I)*SINF(PSI))**2/2.
      RM1=RM1+RM*4.*RG(I)/SQRTF(1.+RG(I)**2)
      FM=(PWB(I)*COSF(PSI))**2+(WA5(I)*SINF(PSI))**2/2.
 3000 FM1=FM1+FM*4.*RG(I)/SQRTF(1.+RG(I)**2)
      DO 3001 I=2,M1
      RM=(P(I)*COSF(PSI))**2+(WW(I)*SINF(PSI))**2/2.
      RM2=RM2+RM*4.*RG(I)/SQRTF(1.+RG(I)**2)
      FM=(PWB(I)*COSF(PSI))**2+(WA5(I)*SINF(PSI))**2/2.
 3001 FM2=FM2+FM*4.*RG(I)/SQRTF(1.+RG(I)**2)
      DO 3002 I=2,M1,2
      RM=(P(I)*COSF(PSI))**2+(WW(I)*SINF(PSI))**2/2.
      RM4=RM4+RM*4.*RG(I)/SQRTF(1.+RG(I)**2)
      FM=(PWB(I)*COSF(PSI))**2+(WA5(I)*SINF(PSI))**2/2.
 3002 FM4=FM4+FM*4.*RG(I)/SQRTF(1.+RG(I)**2)
      FMI=AR*DRB/3.*(FM1+FM2+2.*FM4)
      RMS=AR*DRB/3.*(RM1+RM2+2.*RM4)
```

TABLE V (Continued)

```
      FMI=SQRTF(FMI)
      RMS=SQRTF(RMS)
      WRITE OUTPUT TAPE 3,127
      WRITE OUTPUT TAPE 3,123,R1,R2,R3,R4,PSIDEG
      WRITE OUTPUT TAPE 3,11
      WRITE OUTPUT TAPE 3,3003
      WRITE OUTPUT TAPE 3,3004,RMS,FMI
      WRITE OUTPUT TAPE 3,11
      IF(MGOTO-2)186,487,487
  487 IF(MGOTO-3)187,187,186
  187 WRITE OUTPUT TAPE 3,111
      WRITE OUTPUT TAPE 3,112,T3DEG,T4DEG
      WRITE OUTPUT TAPE 3,11
  186 IF(TPSI)188,189,188
  188 WRITE OUTPUT TAPE 3,173
      WRITE OUTPUT TAPE 3,174,B1,B2,B3,B4,B5,B6
      WRITE OUTPUT TAPE 3,11
      IF(PSI)190,190,189
  189 WRITE OUTPUT TAPE 3,175
      WRITE OUTPUT TAPE 3,176,A1,A2,A3,A4,A5,A6,A7,A8
      WRITE OUTPUT TAPE 3,11
  190 WRITE OUTPUT TAPE 3,10
C
C - - (10) OUTPUT II (DEPENDENT VARIABLES AT THE TABULAR POINTS)
C
      IF(MGOTO-3)450,451,3005
 3005 IF(K3)98,98,402
  451 IF(TPSI)450,452,450
  452 WRITE OUTPUT TAPE 3,140
      DO 255 J=1,N
      DO 255 I=1,M
      W(I,J)=WW(I)*SINF(THETA(J))-P(I)
      WFLEX(I,J)=WA5(I)*SINF(THETA(J))-PWB(I)
      WP(I,J)=W3(I)*SINF(THETA(J))-W1(I)
      U(I,J)=UU(I)*SINF(THETA(J))-Q(I)
  255 V(I,J)=VV(I)*COSF(THETA(J))
      SKIP=1
      GO TO 94
  450 IF(PSI)96,96,23
   23 IF(TPSI)403,95,403
  403 DO 405 I=1,M
      P2(I)=P(I)*COSF(PSI)
      PWB2(I)=PWB(I)*COSF(PSI)
      W2(I)=W1(I)*COSF(PSI)
      FMR(I)=EMR(I)*COSF(PSI)
      FNR(I)=ENR(I)*COSF(PSI)
      Q2(I)=Q1(I)*COSF(PSI)
      FMT(I)=EMT(I)*COSF(PSI)
      FNT(I)=ENT(I)*COSF(PSI)
      Q3(I)=Q(I)*COSF(PSI)
      W4(I)=WW(I)*SINF(PSI)
      WA54(I)=WA5(I)*SINF(PSI)
      W5(I)=W3(I)*SINF(PSI)
      GMR(I)=DMR(I)*SINF(PSI)
      GMT(I)=DMT(I)*SINF(PSI)
      GMRT(I)=DMRT(I)*SINF(PSI)
      QRR(I)=QQR(I)*SINF(PSI)
      QTT(I)=QQT(I)*SINF(PSI)
```

```
      GNR(I)=DNR(I)*SINF(PSI)
      GNT(I)=DNT(I)*SINF(PSI)
      GNRT(I)=DNRT(I)*SINF(PSI)
      U2(I)=UU(I)*SINF(PSI)
      WT(I)=W4(I)+P2(I)-P(I)
      WFLEXT(I)=WA54(I)+PWB2(I)-PWB(I)
      WPT(I)=W5(I)+W2(I)-W1(I)
      UT(I)=U2(I)+Q3(I)-Q(I)
  405 V2(I)=VV(I)*SINF(PSI)
      IF(MGOTO-1)96,796,69
   69 DO 70 J=1,N
      DO 70 I=1,M
      W(I,J)=W4(I)*SINF(THETA(J))+P2(I)
      WFLEX(I,J)=WA54(I)*SINF(THETA(J))+PWB2(I)
      WP(I,J)=W5(I)*SINF(THETA(J))+WZI
      CMR(I,J)=GMR(I)*SINF(THETA(J))+FMR(I)
      CMT(I,J)=GMT(I)*SINF(THETA(J))+FMT(I)
      CMRT(I,J)=GMRT(I)*COSF(THETA(J))
      QR(I,J)=QRR(I)*SINF(THETA(J))+Q2(I)
      QT(I,J)=QTT(I)*COSF(THETA(J))
      CNR(I,J)=GNR(I)*SINF(THETA(J))+FNR(I)
      CNT(I,J)=GNT(I)*SINF(THETA(J))+FNT(I)
      CNRT(I,J)=GNRT(I)*COSF(THETA(J))
      U(I,J)=U2(I)*SINF(THETA(J))+Q3(I)
   70 V(I,J)=V2(I)*COSF(THETA(J))
      SKIP=0
      IF(MGOTO-2)94,94,215
  215 DO 217 J=1,N
      DO 217 I=1,M
      U(I,J)=U(I,J)-Q(I)
      W(I,J)=W(I,J)-P(I)
      WFLEX(I,J)=WFLEX(I,J)-PWB(I)
  217 WP(I,J)=WP(I,J)-W1(I)
      WRITE OUTPUT TAPE 3,140
      SKIP=1
      GO TO 94
   96 IF(PSI)230,230,231
  231 WRITE OUTPUT TAPE 3,138
  230 IF(NORM)240,240,241
  240 WRITE OUTPUT TAPE 3,324
      GO TO 242
  241 WRITE OUTPUT TAPE 3,124
  242 IF(PSI)520,520,521
  520 WRITE OUTPUT TAPE 3,125,(Y(I),P(I),W1(I),ENR(I),ENT(I),I=1,M)
      GO TO 522
  521 WRITE OUTPUT TAPE 3,125,(Y(I),P2(I),W2(I),FNR(I),FNT(I),I=1,M)
  522 WRITE OUTPUT TAPE 3,10
      IF(PSI)232,232,233
  233 WRITE OUTPUT TAPE 3,138
  232 IF(NORM)243,243,244
  243 WRITE OUTPUT TAPE 3,326
      GO TO 245
  244 WRITE OUTPUT TAPE 3,126
  245 IF(PSI)523,523,524
  523 WRITE OUTPUT TAPE 3,125,(Y(I),EMR(I),EMT(I),Q1(I),Q(I),I=1,M)
      GO TO 525
  524 WRITE OUTPUT TAPE 3,125,(Y(I),FMR(I),FMT(I),Q2(I),Q3(I),I=1,M)
  525 WRITE OUTPUT TAPE 3,10
```

TABLE V (Continued)

```
      IF(PSI)900,900,901
  901 WRITE OUTPUT TAPE 3,138
  900 IF(NORM)902,902,903
  902 WRITE OUTPUT TAPE 3,380
      GO TO 904
  903 WRITE OUTPUT TAPE 3,480
  904 IF(PSI)905,905,906
  905 WRITE OUTPUT TAPE 3,925,(Y(I),PWB(I),I=1,M)
      GO TO 907
  906 WRITE OUTPUT TAPE 3,925,(Y(I),PWB2(I),I=1,M)
  907 WRITE OUTPUT TAPE 3,10
      IF(PSI)99,99,95
   95 IF(JPSI)234,235,235
  234 WRITE OUTPUT TAPE 3,138
  235 IF(NORM)246,246,247
  246 WRITE OUTPUT TAPE 3,313
      GO TO 248
  247 WRITE OUTPUT TAPE 3,113
  248 IF(TPSI)527,526,526
  526 WRITE OUTPUT TAPE 3,125,(Y(I),WW(I),W3(I),VV(I),UU(I),I=1,M)
      GO TO 528
  527 WRITE OUTPUT TAPE 3,125,(Y(I),W4(I),W5(I),V2(I),U2(I),I=1,M)
  528 WRITE OUTPUT TAPE 3,10
      IF(TPSI)236,237,237
  236 WRITE OUTPUT TAPE 3,138
  237 IF(NORM)249,249,250
  249 WRITE OUTPUT TAPE 3,314
      GO TO 251
  250 WRITE OUTPUT TAPE 3,114
  251 IF(TPSI)530,529,529
  529 WRITE OUTPUT TAPE 3,125,(Y(I),DMR(I),DMT(I),DMRT(I),QQR(I),I=1,M)
      GO TO 531
  530 WRITE OUTPUT TAPE 3,125,(Y(I),GMR(I),GMT(I),GMRT(I),QRR(I),I=1,M)
  531 WRITE OUTPUT TAPE 3,10
      IF(TPSI)238,239,239
  238 WRITE OUTPUT TAPE 3,138
  239 IF(NORM)252,252,253
  252 WRITE OUTPUT TAPE 3,315
      GO TO 254
  253 WRITE OUTPUT TAPE 3,115
  254 IF(TPSI)533,532,532
  532 WRITE OUTPUT TAPE 3,125,(Y(I),DNR(I),DNT(I),DNRT(I),QQT(I),I=1,M)
      GO TO 534
  533 WRITE OUTPUT TAPE 3,125,(Y(I),GNR(I),GNT(I),GNRT(I),QTT(I),I=1,M)
  534 WRITE OUTPUT TAPE 3,10
      IF(TPSI)908,909,909
  908 WRITE OUTPUT TAPE 3,138
  909 IF(NORM)910,910,911
  910 WRITE OUTPUT TAPE 3,380
      GO TO 912
  911 WRITE OUTPUT TAPE 3,480
  912 IF(TPSI)914,913,913
  913 WRITE OUTPUT TAPE 3,925,(Y(I),WA5(I),I=1,M)
      GO TO 915
  914 WRITE OUTPUT TAPE 3,925,(Y(I),WA54(I),I=1,M)
  915 WRITE OUTPUT TAPE 3,10
      IF(K3)98,98,402
  796 WRITE OUTPUT TAPE 3,140
```

TABLE V (Continued)

```
      IF(NORM)797,797,798
  797 WRITE OUTPUT TAPE 3,327
      GO TO 799
  798 WRITE OUTPUT TAPE 3,328
  799 WRITE OUTPUT TAPE 3,329,(Y(I),WT(I),WFLEXT(I),WPT(I),UT(I),V2(I),
     1I=1,M)
      WRITE OUTPUT TAPE 3,10
      GO TO 99
   94 DO 160 J=1,N
  160 THETR(J)=THETA(J)*RAD
      IF(NORM)601,601,604
  601 IF(SKIP)602,602,603
  602 WRITE OUTPUT TAPE 3,361
      GO TO 607
  603 WRITE OUTPUT TAPE 3,461
      GO TO 607
  604 IF(SKIP)605,605,606
  605 WRITE OUTPUT TAPE 3,161
      GO TO 607
  606 WRITE OUTPUT TAPE 3,261
  607 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
      WRITE OUTPUT TAPE 3,11
      WRITE OUTPUT TAPE 3,FMT2,(Y(I),(W(I,J),J=1,N),I=1,M)
      WRITE OUTPUT TAPE 3,10
      IF(NORM)608,608,611
  608 IF(SKIP)609,609,610
  609 WRITE OUTPUT TAPE 3,362
      GO TO 614
  610 WRITE OUTPUT TAPE 3,462
      GO TO 614
  611 IF(SKIP)612,612,613
  612 WRITE OUTPUT TAPE 3,162
      GO TO 614
  613 WRITE OUTPUT TAPE 3,262
  614 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
      WRITE OUTPUT TAPE 3,11
      WRITE OUTPUT TAPE 3,FMT2,(Y(I),(WFLEX(I,J),J=1,N),I=1,M)
      WRITE OUTPUT TAPE 3,10
      IF(NORM)615,615,618
  615 IF(SKIP)616,616,617
  616 WRITE OUTPUT TAPE 3,363
      GO TO 621
  617 WRITE OUTPUT TAPE 3,463
      GO TO 621
  618 IF(SKIP)619,619,620
  619 WRITE OUTPUT TAPE 3,163
      GO TO 621
  620 WRITE OUTPUT TAPE 3,263
  621 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
      WRITE OUTPUT TAPE 3,11
      WRITE OUTPUT TAPE 3,FMT2,(Y(I),(WP(I,J),J=1,N),I=1,M)
      WRITE OUTPUT TAPE 3,10
      IF(NORM)622,622,625
  622 IF(SKIP)623,623,624
  623 WRITE OUTPUT TAPE 3,364
      GO TO 628
  624 WRITE OUTPUT TAPE 3,464
      GO TO 628
```

TABLE V (Continued)

```
625 IF(SKIP)626,626,627
626 WRITE OUTPUT TAPE 3,164
    GO TO 628
627  WRITE OUTPUT TAPE 3,264
628 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(U(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)629,629,632
629 IF(SKIP)630,630,631
630 WRITE OUTPUT TAPE 3,365
    GO TO 635
631 WRITE OUTPUT TAPE 3,465
    GO TO 635
632 IF(SKIP)633,633,634
633 WRITE OUTPUT TAPE 3,165
    GO TO 635
634 WRITE OUTPUT TAPE 3,265
635 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(V(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(SKIP)599,599,99
599 IF(NORM)267,267,268
267 WRITE OUTPUT TAPE 3,382
    GO TO 269
268 WRITE OUTPUT TAPE 3,582
269 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CMR(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)270,270,271
270 WRITE OUTPUT TAPE 3,366
    GO TO 272
271 WRITE OUTPUT TAPE 3,166
272 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CMT(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)273,273,274
273 WRITE OUTPUT TAPE 3,367
    GO TO 275
274 WRITE OUTPUT TAPE 3,167
275 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CMRT(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)276,276,277
276 WRITE OUTPUT TAPE 3,368
    GO TO 278
277 WRITE OUTPUT TAPE 3,168
278 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(QR(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)279,279,280
279 WRITE OUTPUT TAPE 3,369
    GO TO 281
```

TABLE V (Continued)

```
280 WRITE OUTPUT TAPE 3,169
281 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(QT(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)282,282,283
282 WRITE OUTPUT TAPE 3,370
    GO TO 284
283 WRITE OUTPUT TAPE 3,170
284 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CNR(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)285,285,286
285 WRITE OUTPUT TAPE 3,371
    GO TO 287
286 WRITE OUTPUT TAPE 3,171
287 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CNT(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
    IF(NORM)288,288,289
288 WRITE OUTPUT TAPE 3,372
    GO TO 290
289 WRITE OUTPUT TAPE 3,172
290 WRITE OUTPUT TAPE 3,FMT1,(THETR(J),J=1,N)
    WRITE OUTPUT TAPE 3,11
    WRITE OUTPUT TAPE 3,FMT2,(Y(I),(CNRT(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,10
500 IF(MGOTO-3)99,99,215
 99 IF(K3)98,98,402
101 FORMAT(7I5)
102 FORMAT(3E20.8)
103 FORMAT(4F15.9)
 10 FORMAT(1H1)
 11 FORMAT(////)
111 FORMAT(51H                    THETA3(DEG.)              THETA4(DEG.)////)
112 FORMAT(2F24.2)
113 FORMAT(116H                     R(IN.)                    W(IN.)
  1          OMEGAR                      V(IN.)                    U(IN.)////)
114 FORMAT(121H                     R(IN.)               MR(IN.-LB./IN.)
  1    MTHETA(IN.-LB./IN.)      MRTHETA(IN.-LB./IN.)        QR(IN./LB.)
  2////)
115 FORMAT(121H                     R(IN.)                  NR(IN./LB.)
  1       NTHETA(IN./LB.)        NRTHETA(IN./LB.)        QTHETA(IN./LB.)
  2////)
118 FORMAT(99H  A PARABOLOIDAL SHELL SUBJECTED TO GRAVITY- - -UN-NORMA
  1LIZED RESULTS BY A SHALLOW SHELL ANALYSIS  ////)
119 FORMAT(99H  A PARABOLOIDAL SHELL SUBJECTED TO GRAVITY- - -NORMALIZ
  1ED RESULTS BY A SHALLOW SHELL ANALYSIS     ////)
120 FORMAT(30H  UNDERFLOW OR OVERFLOW       ////)
121 FORMAT(30H  THE MATRIX IS SINGULAR      ////)
122 FORMAT(5F24.5)
123 FORMAT(4F24.5,F18.2)
124 FORMAT(121H                     R(IN.)                    W(IN.)
  1          OMEGAR                   NR(LB./IN.)         NTHETA(LB./IN.)
  1////)
125 FORMAT(F24.4,4F24.8)
```

TABLE V (Continued)

```
 126 FORMAT(121H                  R(IN.)              MR(IN.-LB./IN.)
    1     MTHETA(IN.-LB./IN.)       QR(LB./IN.)           U(IN.)
    1////)
 127 FORMAT(116H                 R1(IN.)                R2(IN.)
    1        R3(IN.)                R4(IN.)              PSI(DEG.)////)
 128 FORMAT(120H     WEIGHT DENSITY(LB./IN.3)    YOUNGS MODULUS(LB./IN.
    12)      THICKNESS(IN.)        FOCAL LENGTH(IN.)     POISSONS RATIO/
    2///)
 129 FORMAT(51H   THE SHELL IS FREE AT R1                      ////)
 130 FORMAT(51H   THE SHELL IS CLAMPED AT R1                   ////)
 131 FORMAT(51H   THE SHELL IS SIMPLY SUPPORTED AT R1          ////)
 132 FORMAT(51H   THE SHELL IS CLAMPED AT R2                   ////)
 133 FORMAT(51H   THE SHELL IS SIMPLY SUPPORTED AT R2          ////)
 134 FORMAT(51H   THE SHELL IS FREE AT R2                      ////)
 136 FORMAT(51H   THE SHELL IS CLOSED AT THE APEX              ////)
 138 FORMAT(51H    THE PORTION OF THE RESULTS INDEPENDENT OF THETA////)
 140 FORMAT(95H THE DISTORTION OF THE SHELL GIVEN BELOW IS MEASURED REL
    1ATIVE TO THAT OF THE FACE-UP POSITION   ////)
 173 FORMAT(7X,2HB1,13X,2HB2,13X,2HB3,13X,2HB4,13X,2HB5,13X,2HB6,////)
 174 FORMAT(6E15.6)
 175 FORMAT (7X,2HA1,13X,2HA2,13X,2HA3,13X,2HA4,13X,2HA5,13X,2HA6,13X,
    12HA7,13X,2HA8,////)
 176 FORMAT(8E15.6)
 205 FORMAT(12A6)
 313 FORMAT(116H                   GAMMA                    W*
    1        OMEGAR*                     V*                   U*  ////)
 314 FORMAT(116H                   GAMMA                MR*
    1       MTHETA*                  MRTHETA*                 QR*  ////)
 315 FORMAT(116H                   GAMMA                  NR*
    1        NTHETA*                 NRTHETA*               QTHETA*////)
 324 FORMAT(116H                   GAMMA                    W*
    1        OMEGAR*                   NR*                  NTHETA*////)
 326 FORMAT(116H                   GAMMA                  MR*
    1        MTHETA*                    QR*                   U*   ////)
 327 FORMAT(121H                GAMMA           W TILDE*         WFLEX
    1TILDE*        OMEGAR TILDE*          U TILDE*           V TILDE*
    2////)
 328 FORMAT(121H              R(IN.)            W TILDE(IN.)        WFLEX TI
    1LDE(IN.)    OMEGAR TILDE(IN.)      U TILDE(IN.)          V TILDE(IN.)
    2////)
 329 FORMAT(F20.4,5F20.8)
 380 FORMAT(51H                    GAMMA                  WFLEX*   ////)
 404 FORMAT(F15.9,I5)
 480 FORMAT(51H                  R(IN.)                WFLEX(IN.) ////)
 925 FORMAT(F24.4,F24.8)
3003 FORMAT(51H               EPISILON              EPISILONFLEX    ////)
3004 FORMAT(2F24.8)
 161 FORMAT(50H               ----DISPLACEMENT W(IN.)----         /////)
 162 FORMAT(50H               ----DISPLACEMENT  WFLEX(IN.)         ////)
 163 FORMAT(50H               ----DISPLACEMENT OMEGAR----         /////)
 164 FORMAT(50H               ----DISPLACEMENT U(IN.)----         /////)
 165 FORMAT(50H               ----DISPLACEMENT V(IN.)----         /////)
 582 FORMAT(60H               ----BENDING MOMENT MR(IN.-LB./IN.)----
    1    /////)
 166 FORMAT(60H               ----BENDING MOMENT MTHETA(IN.-LB./IN.)---
    1-   /////)
 167 FORMAT(60H               ----BENDING MOMENT MRTHETA(IN.-LB./IN.)--
    1--  /////)
```

TABLE V (Continued)

```
  168 FORMAT(50H                  ----TRANSVERSE SHEAR QR(LB./IN.)---/////)
  169 FORMAT(60H                  ----TRANSVERSE SHEAR QTHETA(LB./IN.)----
     1    /////)
  170 FORMAT(60H                  ----STRESS RESULTANT NR(LB./IN.)----
     1    /////)
  171 FORMAT(60H                  ----STRESS RESULTANT NTHETA(LB./IN.)----
     1    /////)
  172 FORMAT(60H                  ----STRESS RESULTANT NRTHETA(LB./IN.)----
     1    /////)
  261 FORMAT(50H              -----DISPLACEMENT W TILDE(IN.)----       /////)
  262 FORMAT(50H              -----DISPLACEMENT WFLEX TILDE(IN.)---- /////)
  263 FORMAT(50H              -----DISPLACEMENT OMEGAR TILDE----       /////)
  264 FORMAT(50H              -----DISPLACEMENT U TILDE(IN.)----       /////)
  265 FORMAT(50H              -----DISPLACEMENT V TILDE(IN.)----       /////)
  361 FORMAT(75H                  ----NORMALIZED DISPLACEMENT W*(DIMENSIONL
     1ESS)----            /////)
  362 FORMAT(75H                  ----NORMALIZED DISPLACEMENT WFLEX* (DIMEN
     1SIONLESS)----       ////)
  363 FORMAT(75H                  ----NORMALIZED DISPLACEMENT OMEGAR*(DIMEN
     1SIONLESS)           /////)
  364 FORMAT(75H                  ----NORMALIZED DISPLACEMENT U*(DIMENSIONL
     1ESS)----            /////)
  365 FORMAT(75H                  ----NORMALIZED DISPLACEMENT V*(DIMENSIONL
     1ESS)----            /////)
  382 FORMAT(75H                  ----NORMALIZED BENDING MOMENT MR*(DIMENSI
     1ONLESS)----         /////)
  366 FORMAT(75H                  ----NORMALIZED BENDING MOMENT MTHETA*(DIM
     1ENSIONLESS)----     /////)
  367 FORMAT(75H                  ----NORMALIZED BENDING MOMENT MRTHETA*(DI
     1MENSIONLESS)----    /////)
  368 FORMAT(75H                  ----NORMALIZED TRANSVERSE SHEAR QR*(DIMEN
     1SIONLESS)----       /////)
  369 FORMAT(75H                  ----NORMALIZED TRANSVERSE SHEAR QTHETA*(D
     1IMENSIONLESS)----   /////)
  370 FORMAT(75H                  ----NORMALIZED STRESS RESULTANT NR*(DIMEN
     1SIONLESS)----       /////)
  371 FORMAT(75H                  ----NORMALIZED STRESS RESULTANT NTHETA*(D
     1IMENSIONLESS)----   /////)
  372 FORMAT(75H                  ----NORMALIZED STRESS RESULTANT NRTHETA*(
     1DIMENSIONLESS)---- /////)
  461 FORMAT(75H              ----NORMALIZED DISPLACEMENT W TILDE*(DIMENSIO
     1NLESS)----          /////)
  462 FORMAT(75H              ----NORMALIZED DISPLACEMENT WFLEX TILDE* (DIM
     1ENSIONLESS)----     /////)
  463 FORMAT(75H              ----NORMALIZED DISPLACEMENT OMEGAR TILDE*(DIM
     1ENSIONLESS)----     /////)
  464 FORMAT(75H              ----NORMALIZED DISPLACEMENT U TILDE*(DIMENSIO
     1NLESS)----          /////)
  465 FORMAT(75H              ----NORMALIZED DISPLACEMENT V TILDE*(DIMENSIO
     1NLESS)----          /////)
      END


*     LABEL
CFCTRL
      FUNCTION FCTRL(INTGER)
```

TABLE V (Continued)

```
D       FCTRL=1.0
        IF (INTGER-1) 2,2,3
      3 DO 4 N=2,INTGER
D       AN=N
D     4 FCTRL=FCTRL*AN
      2 RETURN
        END



*       LABEL
CBER
        FUNCTION BER (X,LONG)
D       PI=3.141592653589793
        IF(X-17.0)5,5,8
D     5 BER=1.0
        DO 16 K=1,LONG
D       AK=K
D       TOPBER= (X/2.0)**(2.0*AK)
D       TERBER=((TOPBER/FCTRL(2*K))**2)*((-1.0)**K)
D    16 BER=BER+TERBER
        GO TO 40
D     8 COEF1=SQRTF(PI/(2.*X))*EXPF(-(X/SQRTF(2.)))
D       COEF2=EXPF(X/SQRTF(2.))/SQRTF(2.*PI*X)
D       ALPHA=X/SQRTF(2.)-PI/8.
D       BETA=X/SQRTF(2.)+PI/8.
D       FIRST=-SINF(BETA)
D       SECOND=COSF(ALPHA)
D       PROD=1.
D       BER=COEF2*SECOND
        DO 53 K=1,LONG
D       AK=K
D       CCPSS=COSF(PI*AK/4.)*COSF(ALPHA)+SINF(PI*AK/4.)*SINF(ALPHA)
D       PROD=PROD*(2.*AK-1.)**2
D       SCFM=PROD/FCTRL(K)
D       SCFM=SCFM/(8.*X)**(AK/2.)
D       SCFM=SCFM/(8.*X)**(AK/2.)
D       SUM2=(SCFM*CCPSS)*COEF2
        IF(X-27.0)60,60,53
D    60 SCFM=SCFM*(-1.)**K
D       SCMCS=-SINF(PI*AK/4.)*COSF(BETA)-COSF(PI*AK/4.)*SINF(BETA)
D       BER=BER-COEF1*FIRST /PI
D       SUM1=       (SCFM*SCMCS)*COEF1/PI
D    53 BER=BER+SUM2-SUM1
     40 RETURN
        END



*       LABEL
CBEI
        FUNCTION BEI (X,LONG)
D       PI=3.141592653589793
        IF(X-17.0)5,5,8
D     5 BEI=0.0
        DO 20 K=1,LONG
```

TABLE V (Continued)

```
D        AK=K
D        TPABEI=(X/2.0)**(2.0*AK)
D        TPBBEI=(X/2.0)**(2.0*AK-2.0)
D        TERBEI=(TPABEI/FCTRL(2*K-1))*(TPBBEI/FCTRL(2*K-1))*((-1.0)**(K-1))
D     20 BEI=BEI+TERBEI
         GO TO 40
D      8 COEF1=SQRTF(PI/(2.*X))*EXPF(-(X/SQRTF(2.)))
D        COEF2=EXPF(X/SQRTF(2.))/SQRTF(2.*PI*X)
D        ALPHA=X/SQRTF(2.)-PI/8.
D        BETA=X/SQRTF(2.)+PI/8.
D        FIRST=COSF(BETA)
D        SECOND=SINF(ALPHA)
D        PROD=1.
D        BEI=COEF2*SECOND
         DO 51 K=1,LONG
D        AK=K
D        CSMSC=COSF(PI*AK/4.)*SINF(ALPHA)-SINF(PI*AK/4.)*COSF(ALPHA)
D        PROD=PROD*(2.*AK-1.)**2
D        SCFM=PROD/FCTRL(K)
D        SCFM=SCFM/(8.*X)**(AK/2.)
D        SCFM=SCFM/(8.*X)**(AK/2.)
D        SUM2=(SCFM*CSMSC)*COEF2
         IF(X-27.0)60,60,51
D     60 SCFM=SCFM*(-1.)**K
D        CCMSS=COSF(PI*AK/4.)*COSF(BETA)-SINF(PI*AK/4.)*SINF(BETA)
D        SUM1=      (SCFM*CCMSS)*COEF1/PI
D        BEI=BEI+COEF1*FIRST/PI
D     51 BEI=BEI+SUM1+SUM2
      40 RETURN
         END



*        LABEL
CAKER
         FUNCTION AKER (X,LONG)
D        PI=3.141592653589793
D        GL2MIG=.1159315156584124
         IF(X-8.89)5,5,8
D      5 SAME=GL2MIG-LOGF(X)
D        PI4TH= PI/4.0
D        AKER=SAME
         DO 25 K=1, LONG
D        AK=K
D        TOPBER= (X/2.0)**(2.0*AK)
D        TERBER=((TOPBER/FCTRL(2*K))**2)*((-1.0)**K)
D        TPABEI=(X/2.0)**(2.0*AK)
D        TPBBEI=(X/2.0)**(2.0*AK-2.0)
D        TERBEI=(TPABEI/FCTRL(2*K-1))*(TPBBEI/FCTRL(2*K-1))*((-1.0)**(K-1))
D        TERM1=SAME*TERBER
D        TERM2=PI4TH*TERBEI
         I=K-1
         IF (I) 7,6,7
D      6 SUM1=1.5
         GO TO 11
D      7 PART=0.0
      57 DO 10 L=I,K
```

TABLE V (Continued)

```
D       AL=L
D   10 PART=PART+(1.0/(AK+AL))
D  110 SUM1=SUM1+PART
D   11 TOP3=((X/2.0)**(2.0*AK))
D   24 TERM3=(((TOP3/FCTRL(2*K))**2)*SUM1*((-1.0)**K))
D   25 AKER=AKER+TERM1+TERM2+TERM3
       GO TO 40
D    8 COEF1=SQRTF(PI/(2.*X))*EXPF(-(X/SQRTF(2.)))
D      ALPHA=X/SQRTF(2.)-PI/8.
D      BETA=X/SQRTF(2.)+PI/8.
D      FIRST=COSF(BETA)
D      PROD=1.
D      AKER=COEF1*FIRST
       DO 50 K=1,LONG
D      AK=K
D      PROD=PROD*((2.*AK-1.)**2)
D      CCMSS=COSF(PI*AK/4.)*COSF(BETA)-SINF(PI*AK/4.)*SINF(BETA)
D      SCFM=PROD/FCTRL(K)
D      SCFM=SCFM/(8.*X)**(AK/2.)
D      SCFM=SCFM/(8.*X)**(AK/2.)*((-1.)**K)
D      SUM1=       (SCFM*CCMSS)*COEF1
D   50 AKER=AKER+SUM1
    40 RETURN
       END




*      LABEL
CAKEI
       FUNCTION AKEI (X,LONG)
D      GL2MIG=.1159315156584124
D      PI=3.141592653589793
       IF(X-8.89)5,5,8
D    5 SAME=GL2MIG-LOGF(X)
D      PI4TH=PI/4.0
D      AKEI=-PI4TH
       DO 25 K=1,LONG
D      AK=K
D      TOPBER= (X/2.0)**(2.0*AK)
D      TERBER=((TOPBER/FCTRL(2*K))**2)*((-1.0)**K)
D      TPABEI=(X/2.0)**(2.0*AK)
D      TPBBEI=(X/2.0)**(2.0*AK-2.0)
D      TERBEI=(TPABEI/FCTRL(2*K-1))*(TPBBEI/FCTRL(2*K-1))*((-1.0)**(K-1))
D      TERM1=SAME*TERBEI
D      TERM2=PI4TH*TERBER
       I=K-1
       IF (I) 86,86,88
D   86 SUM2=1.0
       GO TO 80
D   88 PART=0.0
       DO 94 L=I,K
D      AL=L
D   94 PART=PART+(1.0/(AK+AL-1.0))
D      SUM2=SUM2+PART
D   80 TERM3=SUM2*((-1.0)**(K-1))*(((X/2.0)**(2.0*AK))/FCTRL(2*K-1))*(((X
D     Z/2.0)**(2.0*AK-2.0))/FCTRL(2*K-1))
D   25 AKEI=AKEI+TERM1-TERM2+TERM3
```

TABLE V (Continued)

```
      GO TO 40
D   8 COEF1=SQRTF(PI/(2.*X))*EXPF(-X/SQRTF(2.))
D     ALPHA=X/SQRTF(2.)-PI/8.
D     BETA=X/SQRTF(2.)+PI/8.
D     FIRST=-SINF(BETA)
D     PROD=1.
D     AKEI=COEF1*FIRST
      DO 52 K=1,LONG
D     AK=K
D     SCMCS=-SINF(PI*AK/4.)*COSF(BETA)-COSF(PI*AK/4.)*SINF(BETA)
D     PROD=PROD*((2.*AK-1.)**2)
D     SCFM=PROD/FCTRL(K)
D     SCFM=SCFM/(8.*X)**(AK/2.)
D     SCFM=SCFM/(8.*X)**(AK/2.)*((-1.)**K)
D     SUM1=      (SCFM*SCMCS)*COEF1
D  52 AKEI=AKEI+SUM1
   40 RETURN
      END



*     LABEL
CBRP
      FUNCTION BRP (X,LONG)
D     PI=3.141592653589793
      IF(X-17.0)5,5,8
D   5 BRP=0.0
      DO 65 K=1,LONG
D     AK=K
D     PARTA=((X/2.0)**(2.0*AK))/FCTRL(2*K-1)
D     PARTB=((X/2.0)**(2.0*AK-1.0))/FCTRL(2*K)
D     TERBRP=PARTA*PARTB*((-1.0)**K)
D  65 BRP=BRP+TERBRP
      GO TO 40
D   8 COEF1=-(SQRTF(PI/(2.*X))*EXPF(-X/SQRTF(2.)))
D     COEF2=EXPF(X/SQRTF(2.))/SQRTF(2.*PI*X)
D     ALPHA=X/SQRTF(2.)-PI/8.
D     BETA= X/SQRTF(2.)+PI/8.
D     FIRST=-SINF(ALPHA)
D     SECOND=COSF(BETA)
D     BRP=COEF2*SECOND
D     PROD=1.0
      DO 55 K=1,LONG
D     AK=K
D     SSMCC=-SINF(PI*AK/4.)*SINF(BETA)-COSF(PI*AK/4.)*COSF(BETA)
D     J=K-1
D     AJ=J
D     IF(J)70,70,71
D  70 PROD=1.0
      GO TO 72
D  71 PROD=PROD*(2.*AJ-1.)**2
D  72 SCFM=PROD*(4.*AK**2-1.)/FCTRL(K)
D     SCFM=SCFM/(8.*X)**(AK/2.)
D     SCFM=SCFM/(8.*X)**(AK/2.)
D     SUM2=SCFM*SSMCC*COEF2
      IF(X-27.0)60,60,55
D  60 SCFM=SCFM*(-1.)**K
```

TABLE V (Continued)

```
D        SCPCS=SINF(PI*AK/4.)*COSF(ALPHA)+COSF(PI*AK/4.)*SINF(ALPHA)
D        SUM1=SCFM*SCPCS*FIRST/PI
D        BRP=BRP-COEF1*FIRST/PI
D     55 BRP=BRP-SUM1+SUM2
      40 RETURN
         END




*        LABEL
CBIP
         FUNCTION BIP (X,LONG)
D        PI=3.141592653589753
         IF(X-17.0)5,5,8
D      5 BIP=0.0
          DO 68 K=1,LONG
D        AK=K
         IF (K-1) 66,66,67
D     66 PARTC=(1.0/(X/2.0))/FCTRL(2*K-1)
D        PARTD=((X/2.0)**(2.0*AK))
         GO TO 74
D     67 PARTC=((X/2.0)**(2.0*AK-3.0))/FCTRL(2*K-1)
D     73 PARTD=((X/2.0)**(2.0*AK))/FCTRL(2*K-2)
D     74 TERBIP=PARTC*PARTD*((-1.0)**(K-1))
D     68 BIP=BIP+TERBIP
         GO TO 40
D      8 COEF1=-(SQRTF(PI/(2.*X))*EXPF(-X/SQRTF(2.)))
D        COEF2=EXPF(X/SQRTF(2.))/SQRTF(2.*PI*X)
D        ALPHA=X/SQRTF(2.)-PI/8.
D        BETA=X/SQRTF(2.)+PI/8.
D        FIRST=COSF(ALPHA)
D        SECOND=SINF(BETA)
D        BIP=COEF2*SECOND
D        PROD=1.0
         DO 57 K=1,LONG
D        AK=K
D        SCMCS=SINF(PI*AK/4.)*COSF(BETA)-COSF(PI*AK/4.)*SINF(BETA)
D        J=K-1
D        AJ=J
D        IF(J)70,70,71
D     70 PROD=1.0
         GO TO 72
D     71 PROD=PROD*(2.*AJ-1.)**2
D     72 SCFM=PROD*(4.*AK**2-1.)/FCTRL(K)
D        SCFM=SCFM/(8.*X)**(AK/2.)
D        SCFM=SCFM/(8.*X)**(AK/2.)
D        SUM2=SCFM*SCMCS*COEF2
         IF(X-27.0)60,60,57
D     60 SCFM=SCFM*(-1.)**K
D        SSMCC=SINF(PI*AK/4.)*SINF(ALPHA)-COSF(PI*AK/4.)*COSF(ALPHA)
D        SUM1=SCFM*SSMCC*COEF1/PI
D        BIP=BIP+FIRST*COEF1/PI
D     57 BIP=BIP+SUM1+SUM2
      40 RETURN
         END
```

TABLE V (Continued)

```
*       LABEL
CAKERP
        FUNCTION AKERP(X,LONG)
D       GL2MIG=.1159315156584124
D       PI=3.141592653589793
        IF(X-8.89)5,5,8
D     5 SAME=GL2MIG-LOGF(X)
D       PI4TH=PI/4.0
D       CONST=1.0/X
D       AKERP=-CONST
        DO 25 K=1,LONG
D       AK=K
D       TOPBER= (X/2.0)**(2.0*AK)
D       TERBER=((TOPBER/FCTRL(2*K))**2)*((-1.0)**K)
D       PARTA=((X/2.0)**(2.0*AK))/FCTRL(2*K-1)
D       PARTB=((X/2.0)**(2.0*AK-1.0))/FCTRL(2*K)
D       TERBRP=PARTA*PARTB*((-1.0)**K)
        IF (K-1) 66,66,67
D    66 PARTC=(1.0/(X/2.0))/FCTRL(2*K-1)
D       PARTD=((X/2.0)**(2.0*AK))
        GO TO 74
D    67 PARTC=((X/2.0)**(2.0*AK-3.0))/FCTRL(2*K-1)
D    73 PARTD=((X/2.0)**(2.0*AK))/FCTRL(2*K-2)
D    74 TERBIP=PARTC*PARTD*((-1.0)**(K-1))
D       TERM1=SAME*TERBRP
D       TERM2=CONST*TERBER
D       TERM3=PI4TH*TERBIP
        I=K-1
        IF (I) 7,6,7
D     6 SUM1=1.5
        GO TO 11
D     7 PART=0.0
     57 DO 10 L=I,K
D       AL=L
D    10 PART=PART+(1.0/(AK+AL))
D   110 SUM1=SUM1+PART
D    11 TERM4=SUM1*(2.0*AK)*(((X/2.0)**(2.0*AK))/FCTRL(2*K))*(((X/2.0)**(2
D      Z.0*AK-1.0))/FCTRL(2*K))*((-1.0)**K)
D    25 AKERP=AKERP+TERM1-TERM2+TERM3+TERM4
        GO TO 40
D     8 COEF1=-(SQRTF(PI/(2.*X))*EXPF(-X/SQRTF(2.)))
D       ALPHA=X/SQRTF(2.)-PI/8.
D       BETA=X/SQRTF(2.)+PI/8.
D       FIRST=COSF(ALPHA)
D       AKERP=COEF1*FIRST
D       PROD=1.0
        DO 54 K=1,LONG
D       AK=K
D       SSMCC=SINF(PI*AK/4.)*SINF(ALPHA)-COSF(PI*AK/4.)*COSF(ALPHA)
D       J=K-1
D       AJ=J
        IF(J)70,70,71
D    70 PROD=1.0
        GO TO 72
D    71 PROD=PROD*(2.*AJ-1.)**2
D    72 SCFM=PROD*(4.*AK**2-1.)/FCTRL(K)
```

TABLE V (Continued)

```
D       SCFM=SCFM/(8.*X)**(AK/2.)
D       SCFM=SCFM/(8.*X)**(AK/2.)*(-1.)**K
D        SUM1=SCFM*SSMCC*COEF1
D   54  AKERP=AKERP+SUM1
    40  RETURN
        END




*       LABEL
CAKEIP
        FUNCTION AKEIP(X,LONG)
D       GL2MIG=.1159315156584124
D       PI=3.141592653589793
        IF(X-15.5)5,5,8
D    5  SAME=GL2MIG-LOGF(X)
D       CONST=1.0/X
D       PI4TH=PI/4.0
D       AKEIP=0.0
        DO 25 K=1,LONG
D       AK=K
D       TPABEI=(X/2.0)**(2.0*AK)
D       TPBBEI=(X/2.0)**(2.0*AK-2.0)
D       TERBEI=(TPABEI/FCTRL(2*K-1))*(TPBBEI/FCTRL(2*K-1))*((-1.0)**(K-1))
D       PARTA=((X/2.0)**(2.0*AK))/FCTRL(2*K-1)
D       PARTB=((X/2.0)**(2.0*AK-1.0))/FCTRL(2*K)
D       TERBRP=PARTA*PARTB*((-1.0)**K)
        IF (K-1) 66,66,67
D   66  PARTC=(1.0/(X/2.0))/FCTRL(2*K-1)
D       PARTD=((X/2.0)**(2.0*AK))
        GO TO 74
D   67  PARTC=((X/2.0)**(2.0*AK-3.0))/FCTRL(2*K-1)
D   73  PARTD=((X/2.0)**(2.0*AK))/FCTRL(2*K-2)
D   74  TERBIP=PARTC*PARTD*((-1.0)**(K-1))
D       TERM1=SAME*TERBIP
D       TERM2=CONST*TERBEI
D       TERM3=PI4TH*TERBRP
        I=K-1
        IF (I) 86,86,88
D   86  SUM2=1.0
        GO TO 80
D   88  PART=0.0
        DO 94 L=I,K
D       AL=L
D   94  PART=PART+(1.0/(AK+AL-1.0))
D       SUM2=SUM2+PART
D   80  TERM4=SUM2*(2.0*AK-1.0)*((-1.0)**(K-1))*(((X/2.0)**(2.0*AK))/FCTRL
D      Z(2*K-1))*(((X/2.0)**(2.0*AK-3.0))/FCTRL(2*K-1))
D   25  AKEIP=AKEIP+TERM1-TERM2-TERM3+TERM4
        GO TO 40
D    8  COEF1=-(SQRTF(PI/(2.*X))*EXPF(-X/SQRTF(2.)))
D       ALPHA=X/SQRTF(2.)-PI/8.
D       BETA=X/SQRTF(2.)+PI/8.
D       FIRST=-SINF(ALPHA)
D       PROD=1.0
D       AKEIP=COEF1*FIRST
        DO 56 K=1,LONG
```

TABLE V (Continued)

```
D      AK=K
D      SCPCS=SINF(PI*AK/4.)*COSF(ALPHA)+COSF(PI*AK/4.)*SINF(ALPHA)
D      AJ=J
D      J=K-1
       IF(J)70,70,71
D   70 PROD=1.0
       GO TO 72
D   71 PROD=PROD*(2.*AJ-1.)**2
D   72 SCFM=PROD*(4.*AK**2-1.)/FCTRL(K)
D      SCFM=SCFM/(8.*X)**(AK/2.)
D      SCFM=SCFM/(8.*X)**(AK/2.)*(-1.)**K
D      SUM1=SCFM*SCPCS*COEF1
D   56 AKEIP=AKEIP+SUM1
    40 RETURN
       END
```

1. Input

The input to our program as indicated by step (1) and step (8) in the master flow chart consists of a number of records prestored on machine tape A2.

Record 1 Control Parameters I (4I5)

This card contains four non-negative fixed-point variables.

| M | MU | ML | MC |
|---|---|---|---|

These parameters control the number and spacing of the $\gamma$-tabular points between R3 and R4 (to be defined later under Record 4).

M ($\leqslant 50$) Total number of $\gamma$-tabular points between R3 and R4.

MU Number of (evenly spaced) intervals in the upper edge zone (a region $2(2fh)^{1/2}$ from R4).

ML Number of (evenly spaced) intervals in the lower edge zone (a region $2(2fh)^{1/2}$ from R3).

MC A control parameter to be set equal to 0, 1, or 2.

MC = 0 $\gamma$-tabular points are evenly spaced.

MC = 1 Tabular points are spaced differently (usually denser) only in the upper edge zone. In particular, there will be MU + 1 tabular points in this edge zone. This option is used primarily when the shell is closed at the apex so that no edge effect appears in the lower edge zone.

MC = 2 Tabular points are spaced differently (usually denser) in both edge zones. There are MU + 1 and ML + 1 tabular points in the upper and lower edge zones respectively.

Record 2 Control Parameters II (5I5)

This card contains five non-negative fixed-point variables

| N | NORM | K1 | K2 | K3 |
|---|---|---|---|---|

N ($\leqslant 10$) Number of equally spaced $\Theta$-tabular points between THETA3 and THETA4 (to be defined later under Record 5) along any fixed latitude of the shell. If only the superscripted quantities in (II-85) and (II-86) are desired, N must be set equal to zero and Records 6 and 7 omitted.

NORM Control parameter specifying whether normalized results are desired.

NORM = 0 Normalized results.

NORM > 0 Un-normalized results.

K1 Control parameter specifying the edge constraints at R1 (to be defined later under Record 4).

K1 = 1 Shell is free at R1.

K1 = 2 Shell is clamped at R1.

K1 = 3 Shell is simply supported at R1.

K1 = 4 Shell is closed at the apex.

K2 Control parameter specifying the edge constraints at R2 (to be defined later under Record 4).

K2 = 1 Shell is free at R2.

K2 = 2 Shell is clamped at R2.

K2 = 3 Shell is simply supported at R2.

K3 Different combinations of PSI and MGOTO (see Record 8) may be used for each structure. If more than one such combination is to be run for the same structure, only Record 8 needs to be changed. K3 then specifies the total number of such combinations for the same structure. Set K3 = 0, if the only run wanted is for $\psi = 0$.

Record 3 Material Parameters (3E20.8)

This card contains three E-type floating-point variables.

| RHO | E | PRT |
|---|---|---|

RHO Volume weight density of the shell (lb/in.$^3$).

E Young's modulus (lb/in.$^2$).

PRT Poisson's ratio.

Record 4 Geometrical Parameters I (4F15.9).

This card contains four F-type floating-point variables.

| R1 | R2 | R3 | R4 |
|---|---|---|---|

R1 (in.) Value of r at the lower edge of the shell. If the shell is closed at the apex, then R1 = 0.

R2 (in.) Value of r at the upper edge of the shell.

R3 (in.) Smallest value of r at which stresses and displacements are to be calculated (smallest $\gamma$-tabular point). Generally, this is the same as R1. However, if the shell is closed at the apex, we set R3 > 0 to avoid possible complications arising from calculating the desired quantities at the apex.

R4 (in.) Largest value of r at which the output is to be given (i.e., the largest $\gamma$-tabular point). Generally, this is the same as R2.

Record 5 Geometrical Parameters II (4F15.9)

This card contains four F-type floating-point variables.

| THETA3 | THETA4 | F | H |
|---|---|---|---|

THETA3 Smallest value of $\Theta$ in fractions of $\pi$ at which output is desired (the smallest $\Theta$-tabular point).

THETA4 Largest $\Theta$-tabular point in fractions of $\pi$ at which output is desired.

F Focal length of the paraboloidal surface (in.).

H Shell thickness (in.).

Record 6 Variable Format Statement I (12A6)

This card provides a format statement for the set of $\Theta$-tabular points, for example,

| 19H R(IN.)/THETA(DEG.), NF11.2 |
|---|

Record 7 Variable Format Statement II (12A6)

This card provides a format statement for the dependent variables in the output, for example,

| F19.6, NF11.6 |
|---|

Record 8 Control Parameters III (F15.9, I5)

This card contains one F-type floating-point variable and one fixed-point variable.

| PSI | MGOTO |
|---|---|

PSI The pointing angle $\psi$ in fractions of $\pi$.

MGOTO Control parameter specifying the type of output desired.

MGOTO = 0 Stresses and displacements for symmetric or antisymmetric deformations or the portion of the results independent of $\Theta$ are given in a concise form.

MGOTO = 1 Only the zero-superscripted tilde quantities (and the corresponding $\tilde{w}^{o}_{flex}$ displacement) are given.

MGOTO = 2 Usual set of structural quantities are given.

MGOTO = 3 Only the tilde displacement quantities (and the corresponding $\tilde{w}_{flex}$ displacement) are given.

MGOTO = 4 Only the root-mean-square of the phase error* is given.

*To be discussed in a subsequent chapter.

2. Output

The output to the shallow shell program is arranged in a manner very similar to that of the membrane analysis. The first part of the output reproduces the input to the program. It states the problem, the edge constraints, the various geometrical and material parameters and whether the numerical results have been normalized. The second part of the output gives the value of the physical quantities for the net of tabular points. If $\psi = 0$, the behavior of the shell is axisymmetric. For this case, we have $v = N_{r\theta} = Q_\theta = M_{r\theta} = 0$, while u, w, $\omega_r$, $N_r$, $N_\theta$, $M_r$, $M_\theta$, and $Q_r$ are all functions of $\gamma$ only. The output in this instance is given in a concise form as in the membrane program. For $0 < \psi \leqslant 0.5$, we have several options of output which are very similar to those of the membrane program:

(1) List only the zero-superscripted quantities for the set of $\gamma$-tabular points with the displacement quantities being those relative to the undeformed shell (set MGOTO = 0).

(2) List only the zero-superscripted displacement quantities $\tilde{u}$, $\tilde{v}$, and $\tilde{w}$ relative to those of the face-up position (set MGOTO = 1).

(3) List all the actual stresses and displacements for the entire net of tabular points with the displacement quantities being those relative to the undeformed shell (set MGOTO = 2).

(4) List only the actual displacements for the net of tabular points relative to those of the face-up position (set MGOTO = 3).

(5) List only the root-mean-square of the phase error* (set MGOTO = 4).

3. Operational Information

The program is coded in FORTRAN language for a 32K IBM 7090 (and 7094) and is compiled by a FORTRAN II compiler. It requires 9 FORTRAN functions, which calculate the values of the Thomson functions and their first derivatives using double-precision arithmetic as described in IBM Bulletin J28-6114 "32K 709/7090 FORTRAN: Double-Precision and Complex Arithmetic." All other routines used appear as library routines on the Lincoln Laboratory library tape.

Some of the subscripted variables and their analytical counterparts are presented in Table VI. It should be understood that if normalized results are requested, affected expressions in the third column of Table VI should be replaced by the corresponding starred quantities.

C. EXAMPLES

1. Axisymmetric Deformations

Consider a shell in the face-up position ($\psi = 0°$) which is closed at the apex and simply supported at the upper edge $r_2$ with

*To be discussed in a subsequent chapter.

TABLE VI

SOME SUBSCRIPTED VARIABLES AND THEIR ANALYTICAL COUNTERPARTS

| Subscripted Variable | Dimension | Analytical Expression |
|---|---|---|
| W | 50 × 10 | $w$ |
| WP | 50 × 10 | $\frac{\partial w}{\partial r}$ |
| V | 50 × 10 | $v$ |
| U | 50 × 10 | $u$ |
| CMR | 50 × 10 | $M_r$ |
| CMT | 50 × 10 | $M_\Theta$ |
| CMRT | 50 × 10 | $M_{r\Theta}$ |
| CNR | 50 × 10 | $N_r$ |
| CNT | 50 × 10 | $N_\Theta$ |
| CNRT | 50 × 10 | $N_{r\Theta}$ |
| QR | 50 × 10 | $Q_r$ |
| QT | 50 × 10 | $Q_\Theta$ |
| WFLEX | 50 × 10 | $w_{flex}$ |
| P | 50 | $w^s$ |
| W1 | 50 | $(\frac{\partial w}{\partial r})^s$ |
| Q | 50 | $u^s$ |
| EMR | 50 | $M_r^s$ |
| EMT | 50 | $M_\Theta^s$ |
| ENR | 50 | $N_r^s$ |
| ENT | 50 | $N_\Theta^s$ |
| Q1 | 50 | $Q_r^s$ |
| PWB | 50 | $w_{flex}^s$ |
| WW | 50 | $w^a$ |
| W3 | 50 | $(\frac{\partial w}{\partial r})^a$ |
| VV | 50 | $v^a$ |
| UU | 50 | $u^a$ |
| DMR | 50 | $M_r^a$ |

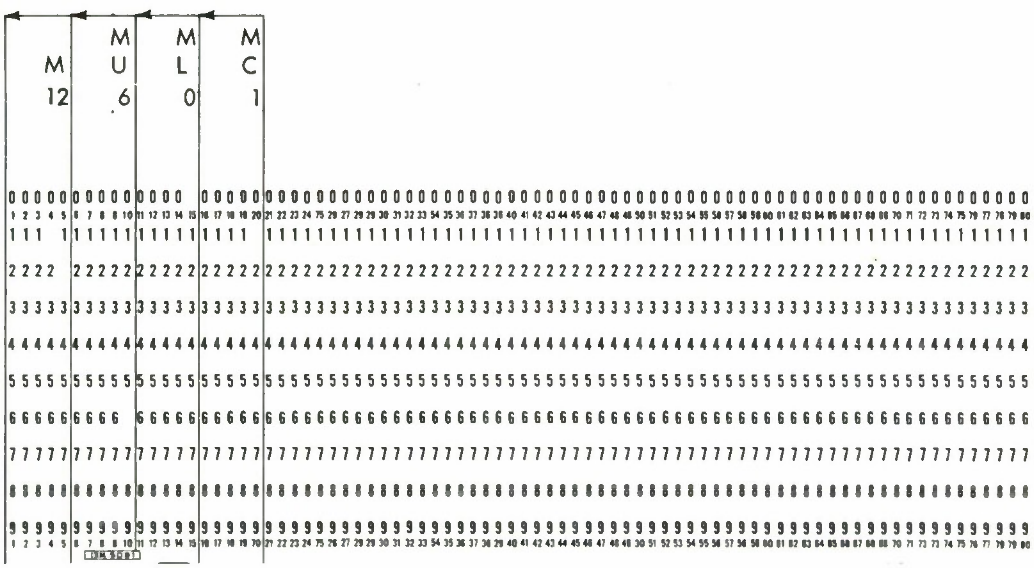
| TABLE VI (Continued) | | |
|---|---|---|
| Subscripted Variable | Dimension | Analytical Expression |
| DMT | 50 | $M_{\Theta}^{a}$ |
| DMRT | 50 | $M_{r\Theta}^{a}$ |
| DNR | 50 | $N_{r}^{a}$ |
| DNT | 50 | $N_{\Theta}^{a}$ |
| DNRT | 50 | $N_{r\Theta}^{a}$ |
| QQR | 50 | $Q_{r}^{a}$ |
| QQT | 50 | $Q_{\Theta}^{a}$ |
| WA5 | 50 | $w_{flex}^{a}$ |
| WT | 50 | $\tilde{w}$ |
| WPT | 50 | $\frac{\partial \tilde{w}}{\partial r}$ |
| UT | 50 | $\tilde{u}$ |
| V2 | 50 | $\tilde{v}$ |
| WFLEXT | 50 | $\tilde{w}_{flex}$ |
| R | 50 | r |
| THETA | 20 | $\Theta$ |
| BBER | 50 | ber |
| BBEI | 50 | bei |
| AAKER | 50 | ker |
| AAKEI | 50 | kei |
| BBRP | 50 | ber' |
| BBIP | 50 | bei' |
| AAKERP | 50 | ker' |
| AAKEIP | 50 | kei' |
| SBER | 50 | ber" |
| SBEI | 50 | bei" |
| SKER | 50 | ker" |
| SKEI | 50 | kei" |
| TBER | 50 | ber''' |
| TBEI | 50 | bei''' |
| TKER | 50 | ker''' |
| TKEI | 50 | kei''' |

$$\text{rho} = 0.1\ \text{lb/in.}^3$$
$$E = 10^7\ \text{lb/in.}^2$$
$$\nu = 0.3$$
$$r_1 = 0\ \text{in.}$$
$$r_2 = 300\ \text{in.}$$
$$f = 450\ \text{in.}$$
$$h = 1\ \text{in.}$$

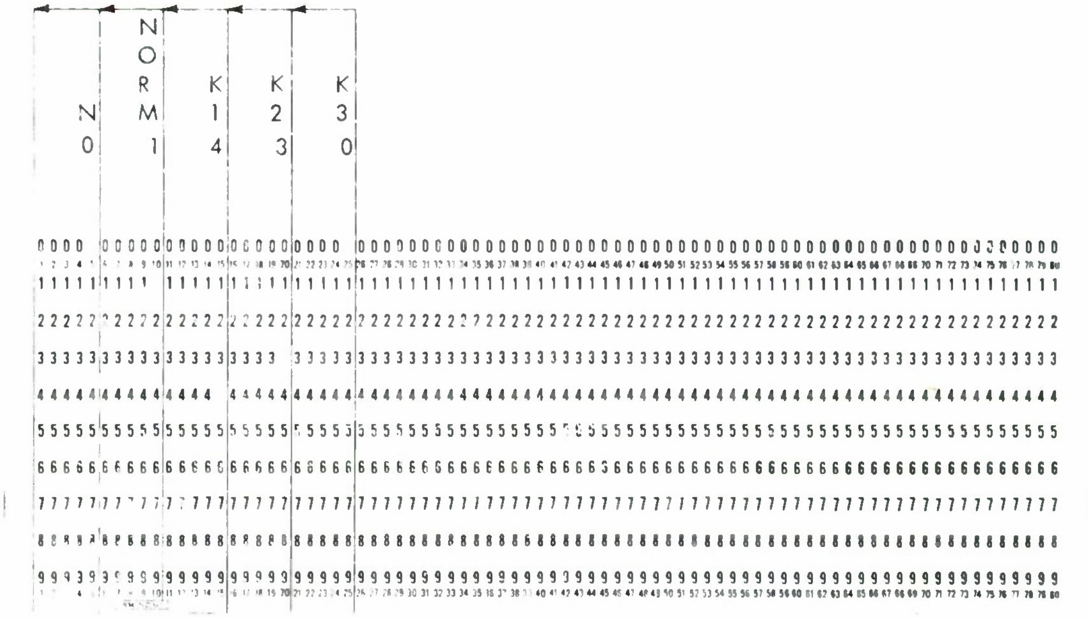
Since the deformation is axisymmetric, we set $N = 0$ and omit Records 6 and 7. For the purpose of illustration, we consider twelve $\gamma$-tabular points with a finer grid in the upper edge zone ($MC = 1$). As the shell is closed at the apex, we avoid calculating the limiting value of the output at the apex by setting $R3 > 0$. The results will not be normalized, so we get the actual physical quantities. Output will be the zero-superscripted quantities for the set of $\gamma$-tabular points, where the displacement quantities are those relative to the undeformed shell.

## INPUT

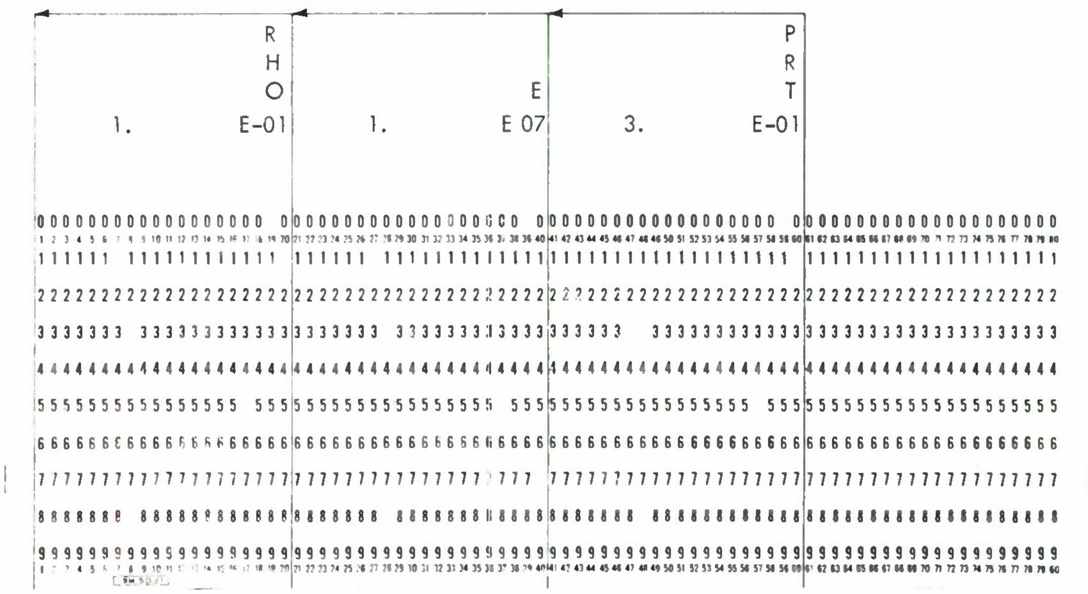
Record 1 Control Parameters I (4I5)



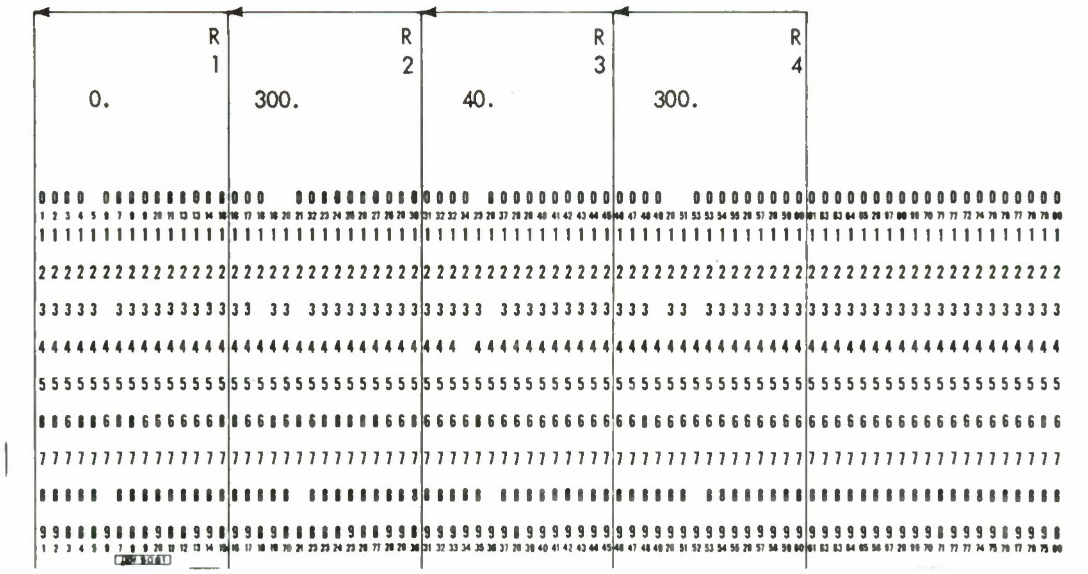
Record 2 Control Parameters II (5I5)



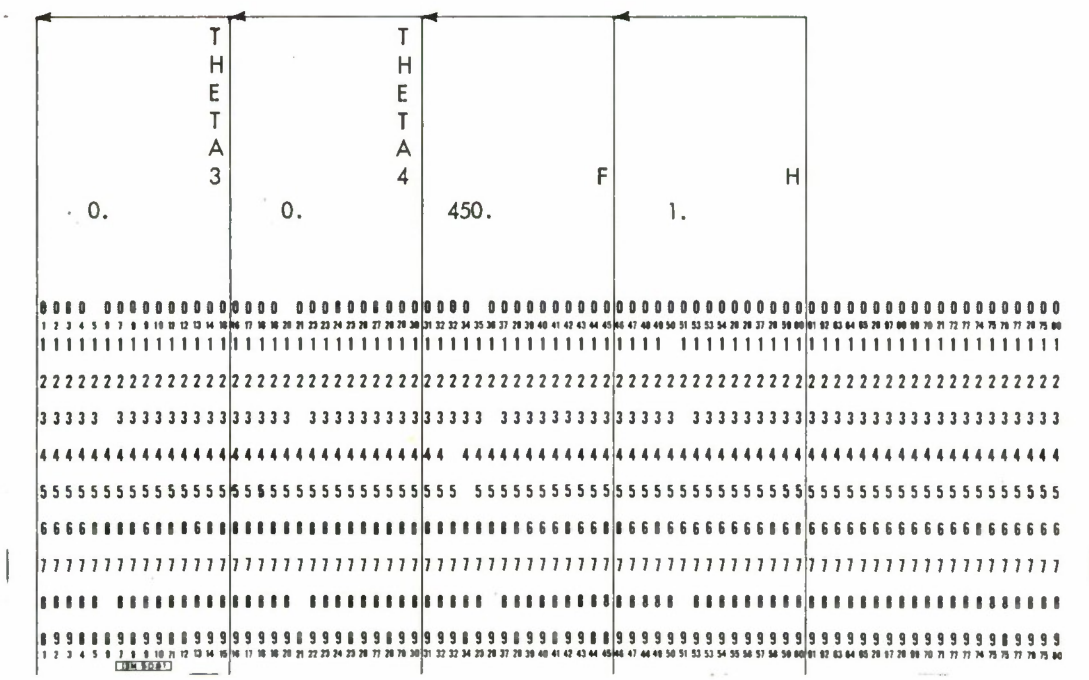

Record 3 Material Parameters (3E20.8)



Record 4 Geometrical Parameters I (4F15.9)

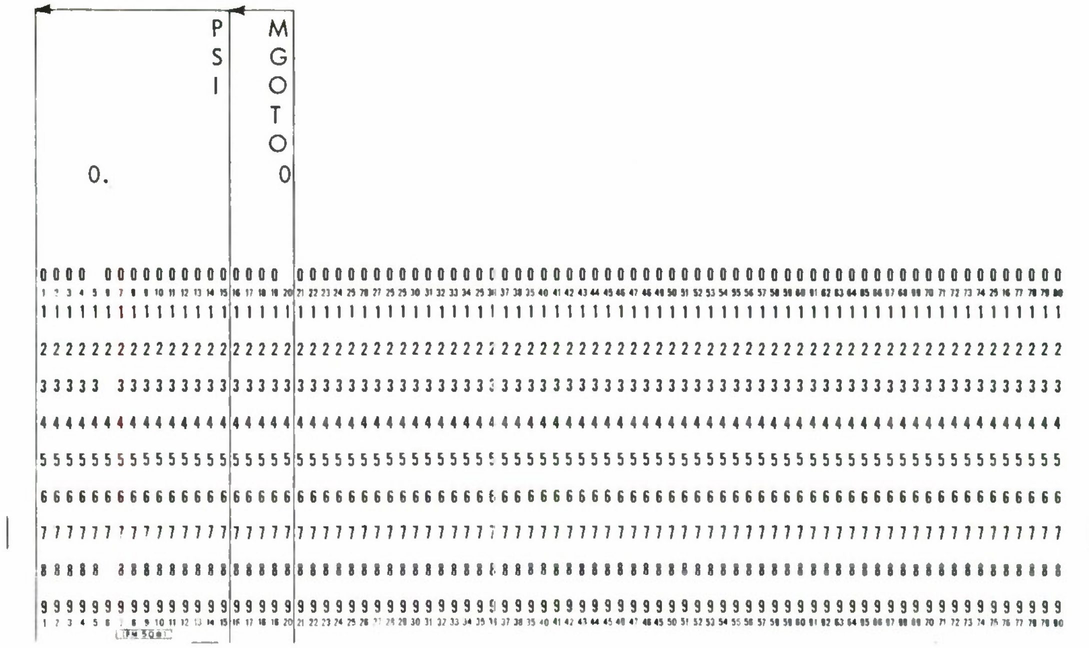



Record 5 Geometrical Parameters II (4F15.9)




Remark: Since N = 0, Records 6 and 7 have been omitted according to input instructions.

Record 8 Control Parameters III (F15.9, I5)




The computer output is presented in Table VII.

2. Unsymmetric Deformations

Consider a shell which is clamped at the lower edge $r_1$, free at the upper edge $r_2$, and oriented in such a way that $\psi \neq 0$. We have

$$
\begin{aligned}
\text{rho} &= 0.1\ \text{lb/in.}^3 \\
E &= 10^7\ \text{lb/in.}^2 \\
\nu &= 0.3 \\
r_1 &= 100\ \text{in.} \\
r_2 &= 300\ \text{in.} \\
f &= 500\ \text{in.} \\
h &= 1\ \text{in.}
\end{aligned}
$$

We now want normalized output for seven $\Theta$-tabular points in the interval $-\pi/2 \leqslant \Theta \leqslant \pi/2$ and five $\gamma$-tabular points in the interval $\gamma_1 \leqslant \gamma \leqslant \gamma_2$. As far as output is concerned, we will list only the actual displacements for the net of tabular points relative to those of the face-up position.

TABLE VII

COMPUTER OUTPUT (AXISYMMETRIC DEFORMATIONS)

```
A PARABOLOIDAL SHELL SUBJECTED TO GRAVITY- - -UN-NORMALIZED RESULTS BY A SHALLOW SHELL ANALYSIS

THE SHELL IS CLOSED AT THE APEX

THE SHELL IS SIMPLY SUPPORTED AT R2

 WEIGHT DENSITY(LB./IN.3)   YOUNGS MODULUS(LB./IN.2)   THICKNESS(IN.)   FOCAL LENGTH(IN.)   POISSONS RATIO

        0.10000            10000000.00000            1.00000          450.00000           0.30000

        R1(IN.)                R2(IN.)               R3(IN.)           R4(IN.)          PSI(DEG.)

        0.                   300.00000             40.00000          300.00000           0.

      EPSILON            EPSILONFLEX

      0.00574002          0.00205270

   B1           B2           B3          B4          B5           B6

0.918840E-07  -0.758887E-08  0.          0.        -0.299815E-02  0.
```

TABLE VII (Continued)

| R(IN.) | W(IN.) | OMEGAR | NR(LB./IN.) | NTHETA(LB./IN.) |
|---|---|---|---|---|
| 40.0000 | -0.00300372 | -0.00000028 | 44.99960995 | 45.00002670 |
| 80.0000 | -0.00302108 | -0.00000057 | 45.0008(921 | 45.00499630 |
| 120.0000 | -0.00304830 | -0.00000072 | 45.00201368 | 44.99522352 |
| 160.0000 | -0.00307680 | -0.00000080 | 44.98898983 | 44.88928223 |
| 200.0000 | -0.00315984 | -0.00000449 | 44.98254728 | 45.2585(631 |
| 240.0000 | -0.00344284 | -0.00000530 | 45.20656967 | 47.49443388 |
| 250.0000 | -0.00345787 | 0.00000350 | 45.29899740 | 47.37836547 |
| 260.0000 | -0.00334433 | 0.00002081 | 45.35578299 | 45.86177236 |
| 270.0000 | -0.00300746 | 0.00004826 | 45.3112(731 | 41.95717382 |
| 280.0000 | -0.00234999 | 0.00008423 | 45.07264709 | 34.67668295 |
| 290.0000 | -0.00132060 | 0.00012051 | 44.53313684 | 23.55685902 |
| 300.0000 | -0.00000000 | 0.0001382( | 43.60551405 | 9.58165598 |

| R(IN.( | MR(IN.-LB./IN.) | MTHETA(IN.-LB./(N.( | QR(LB./IN.( | U((N.) |
|---|---|---|---|---|
| 40.0000 | 0.00868762 | 0.00854930 | 0.00001731 | -0.00000724 |
| 80.0000 | 0.00825081 | 0.00851767 | -0.00007290 | -0.00001451 |
| 120.0000 | 0.00103077 | 0.00537544 | -0.00026862 | -0.00002178 |
| (60.0000 | 0.01908101 | 0.00992526 | 0.00(95726 | -0.00002877 |
| 200.0000 | 0.(7224405 | 0.07038863 | 0.00307818 | -0.00003580 |
| 240.0000 | -0.49331048 | -0.12955520 | -0.05500547 | -0.00004995 |
| 250.0000 | -1.161800(9 | -0.36022269 | -0.08305497 | -0.00005504 |
| 260.0000 | -2.06254309 | -0.68547198 | -0.10278189 | -0.00005916 |
| 270.0000 | -3.01600832 | -1.05375478 | -0.09336543 | -0.00005987 |
| 280.0000 | -3.58491150 | -1.32615966 | -0.02260149 | -0.00005340 |
| 290.0000 | -2.97528461 | -1.23889700 | 0.15043341 | -0.00003497 |
| 300.0000 | -0.00000042 | -0.38392065 | 0.46482836 | -0. |

| R(IN.) | WFLEX(IN.( |
|---|---|
| 40.0000 | -0.00000557 |
| 80.0000 | -0.00002292 |
| 120.0000 | -0.00005015 |
| 160.0000 | -0.00007864 |
| 200.0000 | -0.00016169 |
| 240.0000 | -0.00044469 |
| 250.0000 | -0.0004597( |
| 260.0000 | -0.00034618 |
| 270.0000 | -0.00000930 |
| 280.0000 | 0.00064816 |
| 290.0000 | 0.00167754 |
| 300.0000 | 0.002998(5 |

TABLE VIII

COMPUTER OUTPUT (UNSYMMETRIC DEFORMATIONS)

```
A PARABOLOIDAL SHELL SUBJECTED TO GRAVITY- - -NORMALIZED RESULTS BY A SHALLOW SHELL ANALYSIS

 THE SHELL IS CLAMPED AT R1

 THE SHELL IS FREE AT R2

   WEIGHT DENSITY(LB./IN.3)    YOUNGS MODULUS(LB./IN.2)      THICKNESS(IN.)      FOCAL LENGTH(IN.)    POISSONS RATIO

               0.10000          10000000.00000              1.00000              500.00000             0.30000

               R1(IN.)                R2(IN.)               R3(IN.)               R4(IN.)            PSI(DEG.)

             100.00000              300.00000             100.00000             300.00000             30.00

             EPISILON              EPISILONFLEX

             0.02580583              1.64701623

              THETA3(DEG.)            THETA4(DEG.)

                 -90.00                   90.00

     B1           B2            B3           B4           B5           B6

-0.527009E-09 -0.799029E-10  0.418463E 01  0.650382E 01 -0.455703E-01 -0.148698E 01

     A1           A2            A3           A4           A5           A6           A7           A8

-0.765727E-09 -0.490867E-09 -0.323470E-01 -0.289259E 01  0.336288E-01 -0.352988E-02  0.192405E 01 -0.960529E-01
```

TABLE VIII (Continued)

THE DISTORTION OF THE SHELL GIVEN BELOW IS MEASURED RELATIVE TO THAT OF THE FACE-UP POSITION

----NORMALIZED DISPLACEMENT W TILDE•(DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | 0.000000 | 0.000000 | 0.000000 | -0.000000 | -0.000000 | -0.000000 | -0.000000 |
| 0.1500 | 1.826221 | 1.658666 | 1.200895 | 0.575569 | -0.049758 | -0.507528 | -0.675084 |
| 0.2000 | 2.522494 | 2.268085 | 1.573026 | 0.623557 | -0.325911 | -1.020970 | -1.275379 |
| 0.2500 | 3.031448 | 2.707404 | 1.822098 | 0.612748 | -0.596603 | -1.481909 | -1.805953 |
| 0.3000 | 3.561562 | 3.166764 | 2.088154 | 0.614746 | -0.858663 | -1.937272 | -2.332071 |

----NORMALIZED DISPLACEMENT WFLEX TILDE• (DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.20 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | -1.625106 | -1.489178 | -1.117816 | -0.610526 | -0.103236 | 0.268126 | 0.404054 |
| 0.1500 | -0.306174 | -0.269838 | -0.170566 | -0.034957 | 0.100651 | 0.199923 | 0.236259 |
| 0.2000 | -0.117191 | -0.099745 | -0.052080 | 0.013031 | 0.078143 | 0.125807 | 0.143254 |
| 0.2500 | -0.115527 | -0.099751 | -0.056652 | 0.002222 | 0.061096 | 0.104195 | 0.119970 |
| 0.3000 | -0.092703 | -0.079717 | -0.044241 | 0.004220 | 0.052601 | 0.088157 | 0.101142 |

----NORMALIZED DISPLACEMENT OMEGAR TILDE•(DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | -0.000001 | -0.000001 | -0.000001 | -0.000001 | -0.000001 | -0.000001 | -0.000001 |
| 0.1500 | 1.880566 | 1.797026 | 1.568792 | 1.257019 | 0.945245 | 0.717011 | 0.633472 |
| 0.2000 | 0.205508 | 0.162073 | 0.043406 | -0.118697 | -0.280799 | -0.399467 | -0.442902 |
| 0.2500 | 0.335796 | 0.291021 | 0.168694 | 0.001591 | -0.165512 | -0.287840 | -0.332615 |
| 0.3000 | 0.343544 | 0.298932 | 0.177049 | 0.010553 | -0.155942 | -0.277825 | -0.322437 |

TABLE VIII (Continued)

----NORMALIZED DISPLACEMENT U TILDE*(DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | -0.000000 | -0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.1500 | 0.076307 | 0.070276 | 0.053799 | 0.031291 | 0.008783 | -3.007694 | -0.013725 |
| 0.2000 | 0.206485 | 0.180653 | 0.139932 | 0.073379 | 0.006826 | -0.041894 | -0.059727 |
| 0.2500 | 0.353545 | 0.320061 | 0.231567 | 0.109589 | -0.012388 | -3.101602 | -0.134366 |
| 0.3000 | 0.523242 | 0.472295 | 0.333104 | 0.142965 | -0.047173 | -0.186364 | -0.237312 |

----NORMALIZED DISPLACEMENT V TILDE*(DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | -0. | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0. |
| 0.1500 | -0. | 0.054314 | 0.094074 | 0.108627 | 0.094074 | 3.054314 | 0. |
| 0.2000 | -0. | 8.106510 | 0.184401 | 0.213020 | 0.184481 | 0.106510 | 0. |
| 0.2500 | -0. | 0.163862 | 0.283817 | 0.327724 | 0.283817 | 0.163862 | 0. |
| 0.3000 | -0. | 0.229297 | 0.397155 | 0.458595 | 0.397155 | 0.229297 | 0. |

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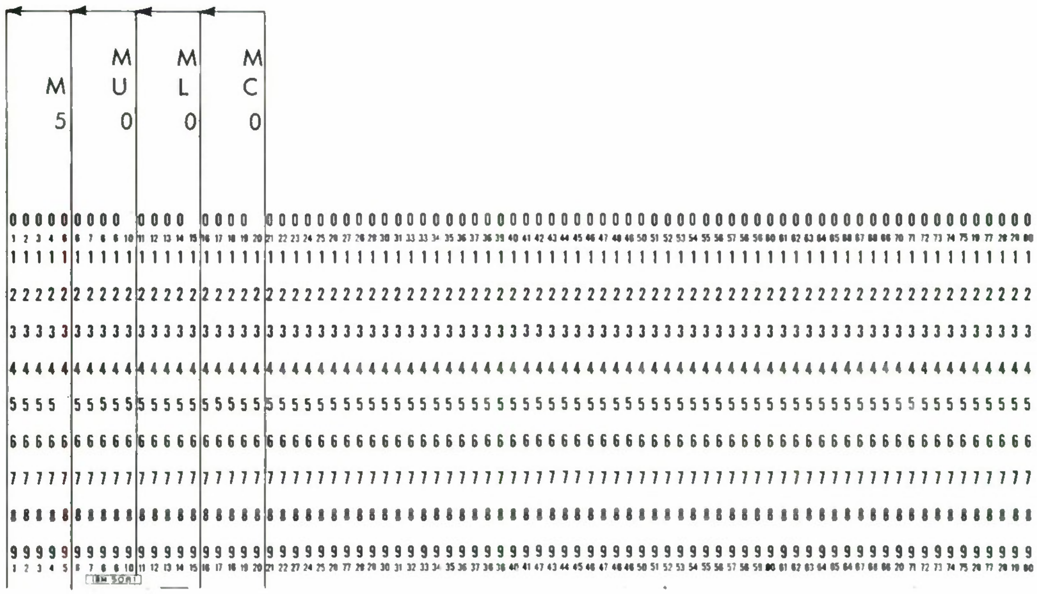


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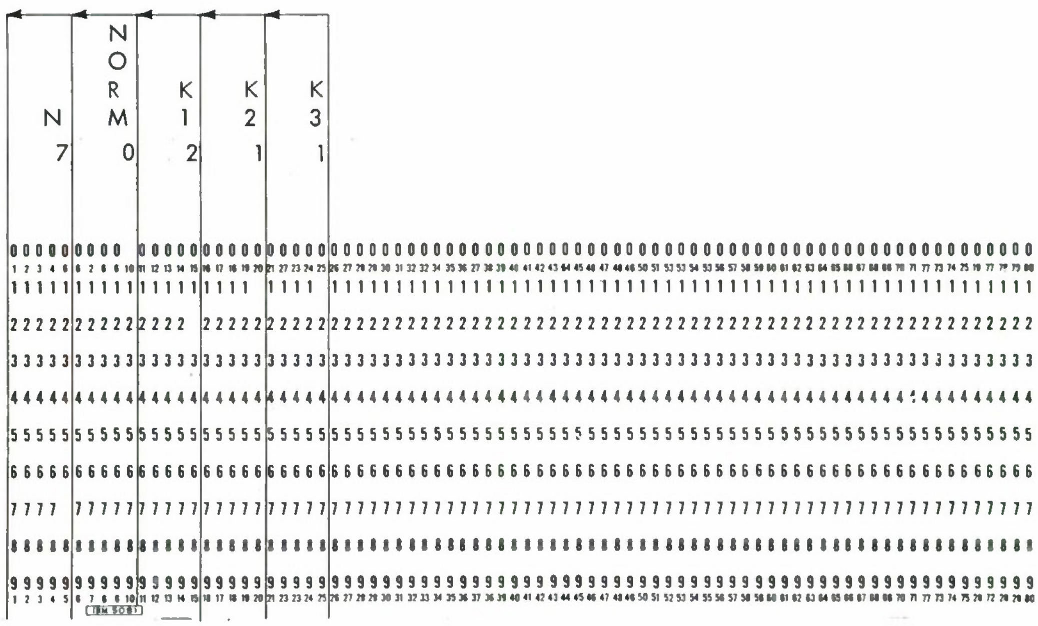

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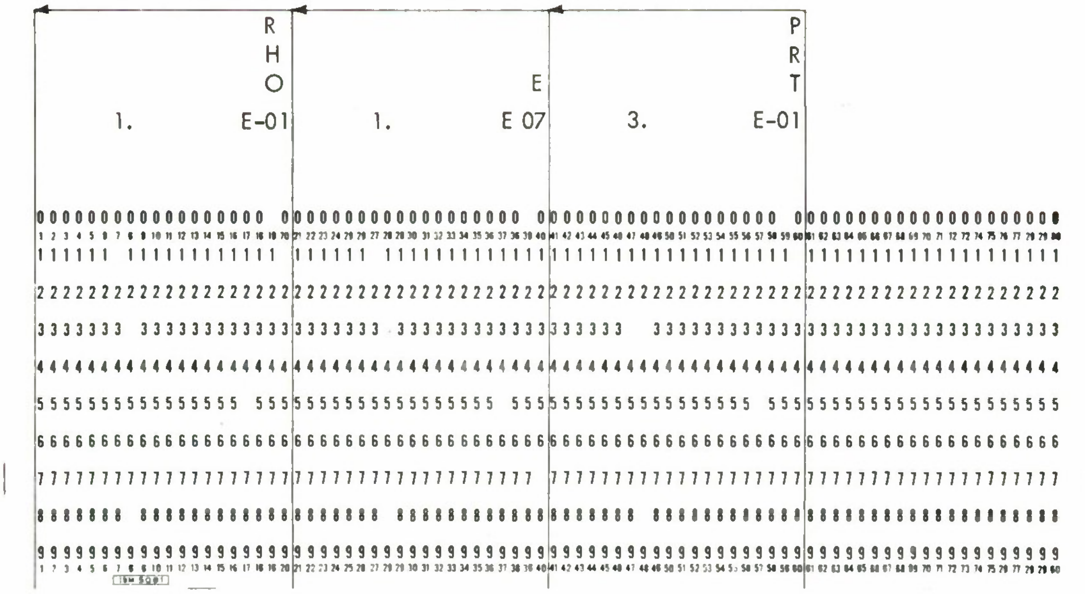


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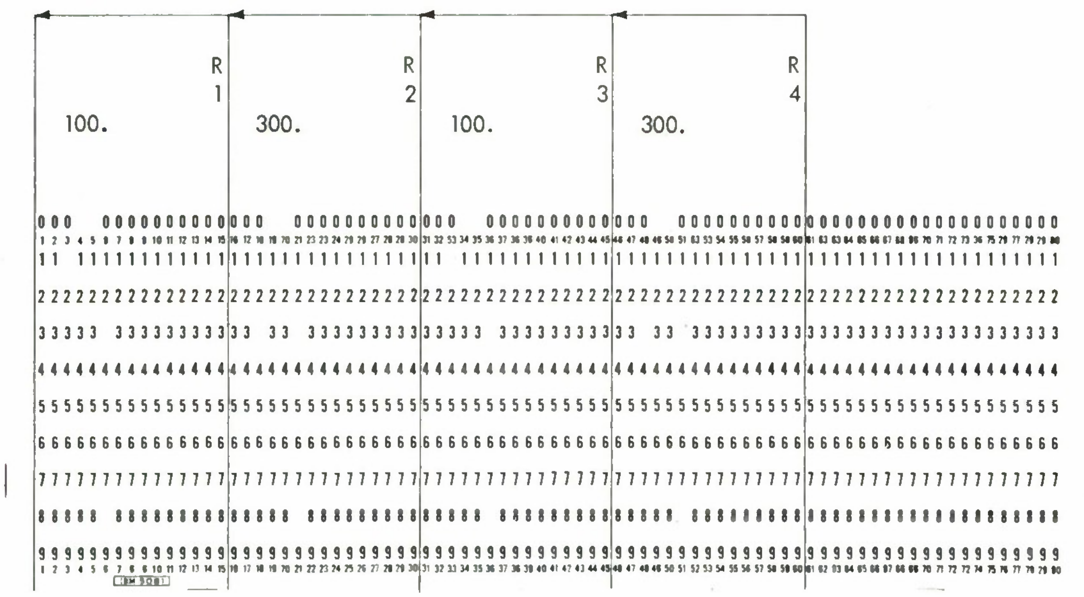

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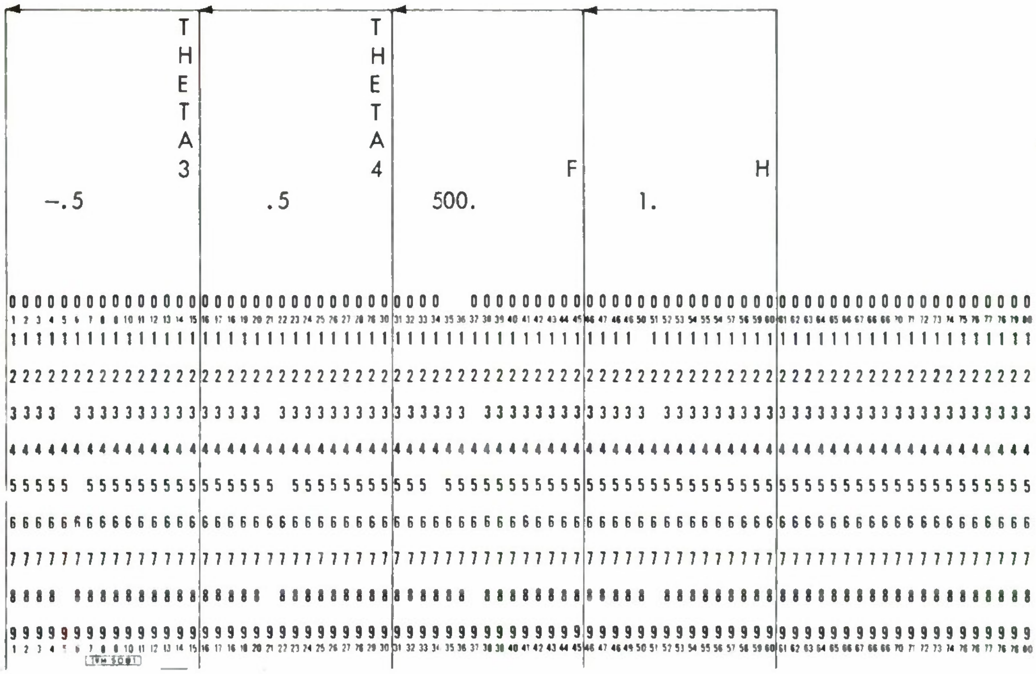


Record 6     Variable Format Statement I-FMT1-(12A6)

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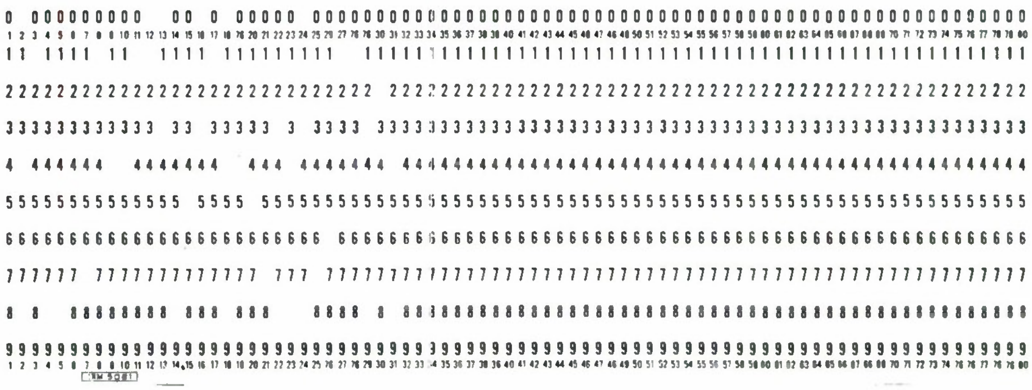

Record 7 Variable Farmat Statement II-FMT2-(12A6)

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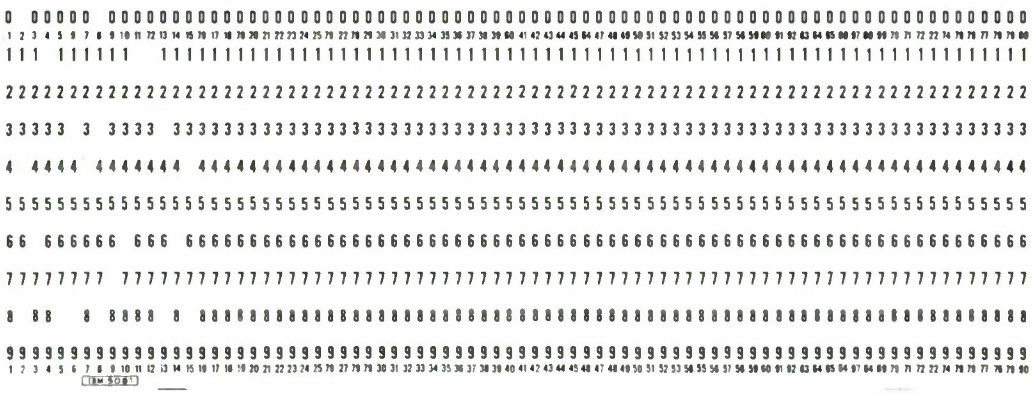

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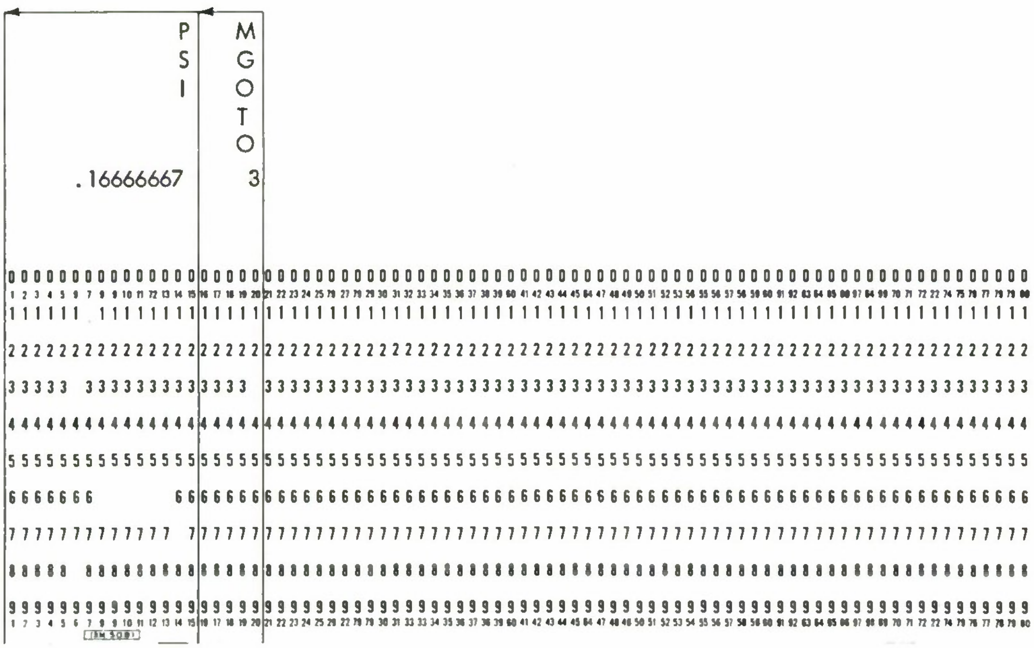


The computer output is presented in Table VIII.

## REFERENCES


1. J. W. Mar and F. Y. M. Wan, "Distortions and Stresses of Paraboloidal Surface Structures," Group Report 71G-1-I, Lincoln Laboratory, M.I.T. (9 January 1962), DDC 296298, H-476.
2. ______________________, "Distortions and Stresses of Paraboloidal Surface Structures. Part II. The Membrane Behavior," Group Report 71G-1-II, Lincoln Laboratory, M.I.T. (4 January 1963), DDC 296491, H-477.
3. ______________________, "Distortions and Stresses of Paraboloidal Surface Structures. Part III," Group Report 71G-1-III, Lincoln Laboratory, M.I.T. (15 August 1963), DDC 417081, H-535.
4. ______________________, "Distortions and Stresses of Paraboloidal Surface Structures," Group Report 71G-1 Part IV, Lincoln Laboratory, M.I.T. (29 April 1964), DDC 600125, H-579.
5. F. Y. M. Wan, "Membrane and Bending Stresses in Shallow, Spherical Shells," Technical Report 317, Lincoln Laboratory, M.I.T. (11 August 1964).
6. H. H. Lowell, "Tables of the Bessel-Kelvin Functions ber-bei-ker-kei, and Their Derivatives for Argument Range 0(0.01) 107.50," NASA Tech. Report R-32, 1959.


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13. ABSTRACT

The primary design requirement of a high-performance antenna is that the reflecting surface remain paraboloidal. For an antenna housed in a radome, strength considerations play a minor design role. Therefore, the antenna must have adequate structural stiffness accompanied by minimum weight. The basic structural components of the antenna are paraboloidal panels, which, when joined together, form a surface of revolution. Such a structural configuration, if properly fabricated, can be considered as a shell. Shell structures derive many of their attractive features from their two-dimensional surface nature, which brings with it geometrical complications to the strain-deflection relations and the equilibrium equations. Although the deflections of trusses, beams, and space frameworks are well understood, well documented, and easily obtained, this is not true for shells. In fact, shell behavior is currently the major topic of study in the structural mechanics field. The available solutions for even simple loadings of simple shells are of such a form that numerical results are not easily obtained. For these reasons, Lincoln Laboratory has been actively studying the deformations of paraboloidal shells. This user's manual will describe the capability, potentiality, and idiosyncrasies of the various LLAPS computer programs which are products of the above study.

14. KEY WORDS

paraboloidal shells antenna design computer program

Rev. 17 November 1965

ESD-TDR-64-578



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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

# LINCOLN LABORATORY ANALYSES OF PARABOLOIDAL SHELLS (LLAPS)

## USER'S MANUAL



LINCOLN MANUAL 60

16 NOVEMBER 1964


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LEXINGTON MASSACHUSETTS

TABLE OF CONTENTS

LINCOLN LABORATORY ANALYSES
OF PARABOLOIDAL SHELLS
(LLAPS)

USER'S MANUAL

# I. INTRODUCTION

The quest for more and more precise radars in the ultra-high frequency régime has imposed stringent requirements on the structural behavior of large antennas. The tolerance requirements for an antenna surface at 10,000 Mcps, for instance, are generally set at about 1/16 of the operating wavelength. At a frequency of 10,000 Mcps, this is 3/16 of a centimeter or 0.074 inch. Such a tolerance on the permissible distortions of a structure which may be a hundred feet or more in over-all size, and which assumes different orientations with respect to the direction of gravity, requires an extremely high degree of sophistication in analysis, design, and construction.

Structures such as bridges, buildings, and flight vehicles are designed mainly by strength considerations, although flight vehicles must also have a certain minimum stiffness in order to avoid aero-elastic difficulties. Machine tools must possess great stiffness, but machine tools are generally compact and weight limitations are relatively unimportant. On the other hand, the primary design requirement of a high-performance antenna is that the reflecting surface remain paraboloidal and, in the case of an antenna housed in a radome, strength considerations play a minor role in the design. The antenna must therefore have adequate structural stiffness but, since the predominant loads are its own dead weight, the structural stiffness should be accompanied by minimum weight; that is, the antenna design should maximize the ratio of structural stiffness to weight.

The basic structural components of the antenna are paraboloidal panels which, when joined together, form a surface of revolution. Such a structural configuration, if properly fabricated, can be considered as a shell. The calculation of stresses for the strength design of a shell can be achieved with a fair degree of confidence, since confidence in the integrity of a structure can be obtained by increasing the factor of safety, i.e., by putting more material into the structure. Such a course of action may be self-defeating in an antenna which has only to resist its own dead weight. Moreover, the determination of the shape of the antenna surface must be precise, and cannot be approached with the same philosophy which is attendant to a strength design, i.e., hidden under a factor of safety.

Shell structures derive many of their attractive features from their two-dimensional surface nature. This two-dimensional nature brings with it geometrical complications to the strain-deflection relations and the equilibrium equations. Although the deflections of trusses, beams, and space frameworks are well understood, well documented, and easily obtained, the exact opposite is true for shells, as evidenced by the fact that shell behavior is currently the major topic of study in the field of structural mechanics. The available solutions for even simple

loadings of simple shells are of such a form that numerical results are not easily obtained. For these reasons, Lincoln Laboratory has been actively studying the deformations of paraboloidal shells.[1-5]

This user's manual will describe the capability, potentiality, and idiosyncrasies of the various LLAPS (Lincoln Laboratory Analyses of Paraboloidal Shells) computer programs which are products of the above study. These programs are all directed at the deflection problem of antennas although the force and moment resultants are also available. This manual is open ended, and additions will be made as new developments are completed.

Accepted for the Air Force
Stanley J. Wisniewski
Lt Colonel, USAF
Chief, Lincoln Laboratory Office

## II. FORMULATION OF PROBLEM

### A. GEOMETRY OF SHELL

The middle surface of the antenna is a paraboloid of revolution whose geometry is described by the following figures and formulas. A point o on the middle surface is located by rectangular Cartesian coordinates $y_1$, $y_2$, $y_3$ or by circular cylindrical coordinates r, $\theta$, $y_3$ (Fig. 1).

$$y_1 = r \cos\theta \tag{II-1}$$

$$y_2 = r \sin\theta \tag{II-2}$$

$$y_3 = \frac{r^2}{4f} \tag{II-3}$$

where f is the focal length of the parabola (Fig. 2). Let

$$\gamma = \frac{r}{2f} \quad . \tag{II-4}$$

Then the slope of the parabola (and therefore the slope of a meridian of the paraboloidal surface) is

$$\frac{dy_3}{dr} = \frac{r}{2f} = \gamma \quad . \tag{II-5}$$

An element of arc length along a meridian (see Fig. 3) is

$$ds_r = 2f\sqrt{1 + \gamma^2}\, d\gamma \quad . \tag{II-6}$$

An element of arc length along a latitude (see Fig. 3) is

$$ds_\theta = 2f\gamma d\theta \quad . \tag{II-7}$$

An element of surface area on the middle surface is

$$dA = 4f^2\gamma\sqrt{1 + \gamma^2}\, d\gamma d\theta \quad . \tag{II-8}$$

A point p in the shell is located by orthogonal coordinates r, $\theta$, $\xi$ (see Fig. 3). [$\xi$ is perpendicular to the middle surface (see Fig. 3) and is positive inward, h is the thickness of shell.]

Let $\overline{ed}$ be of unit length, then the directional cosines of $\xi$, that is, the orientation of $\xi$ (see Fig. 4), are given by

$$\overline{ec} = \frac{\gamma \cos\theta}{\sqrt{1 + \gamma^2}} \quad \text{(Note it is in the negative } y_1 \text{ direction.)} \tag{II-9}$$

$$\overline{eb} = \frac{\gamma \sin\theta}{\sqrt{1 + \gamma^2}} \quad \text{(Note it is in the negative } y_2 \text{ direction.)} \tag{II-10}$$

and

$$\overline{ad} = \frac{1}{\sqrt{1 + \gamma^2}} \quad . \tag{II-11}$$

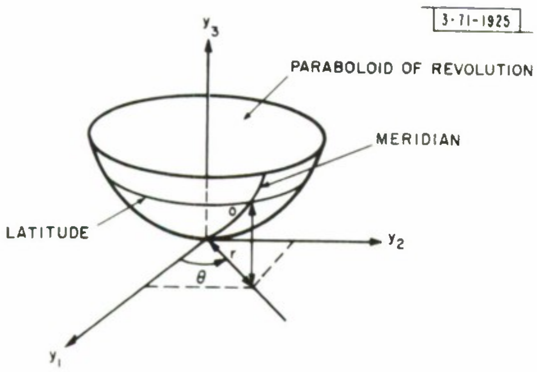



Fig. 1. Paraboloid of revolution.

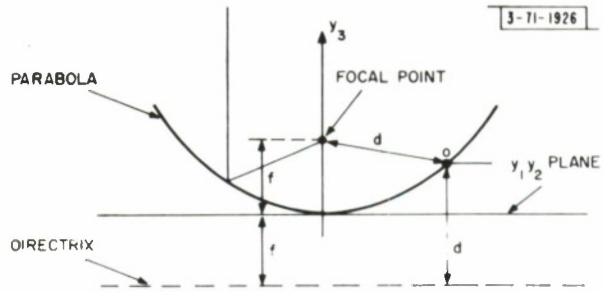



Fig. 2. Focal length of parabola.

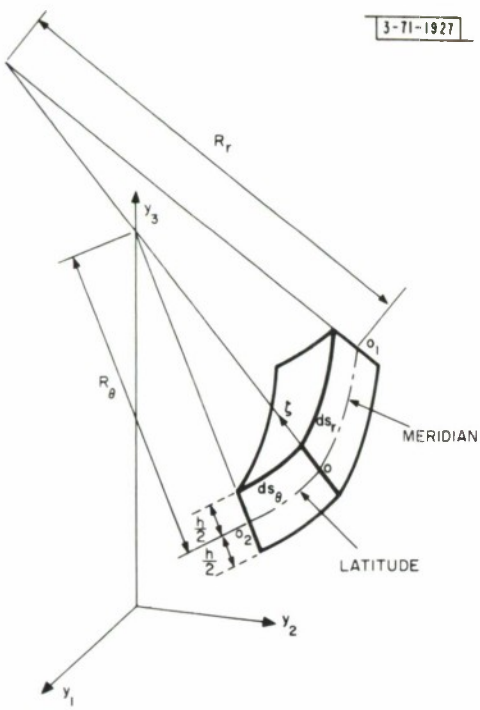



Fig. 3. Element of shell.

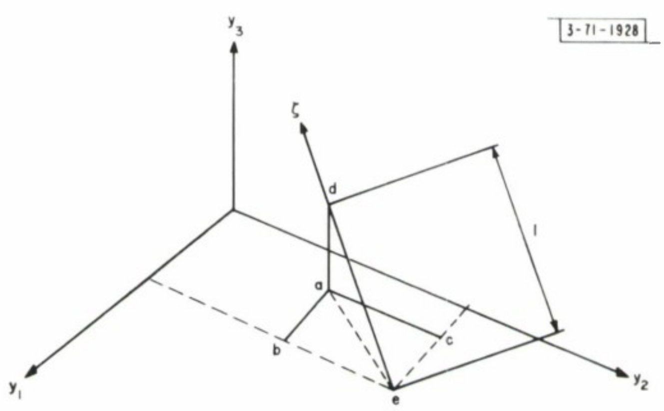



Fig. 4. Directional cosines.

The principal radius of curvature along a meridian (see Fig. 3) is

$$R_r = 2f(1 + \gamma^2)^{3/2} \tag{II-12}$$

and the principal radius of curvature along a latitude (see Fig. 3) is

$$R_\Theta = 2f\sqrt{1 + \gamma^2} \tag{II-13}$$

The boundaries or edges of shell are described by (see Fig. 5)

$$r = r_1 = \text{the radius of the inner edge} \tag{II-14}$$

$$r = r_2 = \text{the radius of the outer edge} \quad . \tag{II-15}$$

The corresponding value of $\gamma$ at the edges will be denoted by $\gamma_1$ and $\gamma_2$, respectively. The directional cosines of a unit vector tangent to a meridian (see Fig. 6) are

$$y_1 \text{ component} \quad \frac{\cos\Theta}{\sqrt{1 + \gamma^2}} \tag{II-16}$$

$$y_2 \text{ component} \quad \frac{\sin\Theta}{\sqrt{1 + \gamma^2}} \tag{II-17}$$

$$y_3 \text{ component} \quad \frac{\gamma}{\sqrt{1 + \gamma^2}} \quad . \tag{II-18}$$

The directional cosines of a unit vector tangent to a latitude (see Fig. 6) are

$$y_1 \text{ component} \quad -\sin\Theta \tag{II-19}$$

$$y_2 \text{ component} \quad \cos\Theta \tag{II-20}$$

$$y_3 \text{ component} \quad 0 \quad . \tag{II-21}$$

Finally, we define a pointing angle $\psi$ to be the angle between the axis of revolution of the paraboloidal surface and the direction of gravity (see Fig. 7).

## B. STRUCTURAL PARAMETERS OF SHELL

The parameters which describe the structural behavior of the shell are the middle-surface displacements, strains, rotations, curvatures, stresses, force resultants, transverse shear resultants, and moment resultants. These will now be defined.

Let

u = displacement of point o along tangent to meridian at point o (see Fig. 8) (II-22)

v = displacement of point o along tangent to latitude at point o (see Fig. 8) (II-23)

w = displacement of point o along normal at point o (see Fig. 8). (II-24)

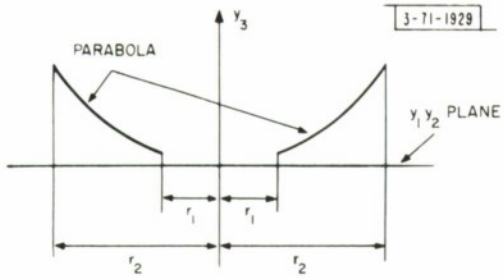



Fig. 5. Edges of shell.

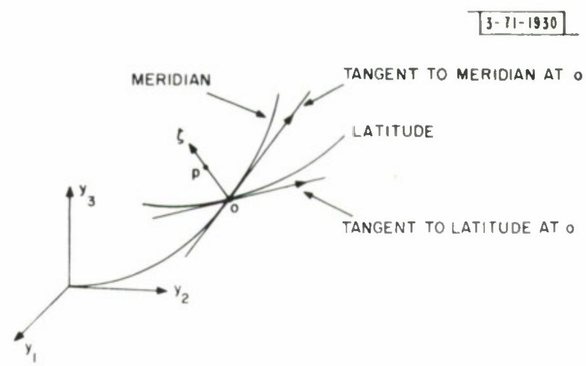



Fig. 6. Base vector and midsurface normal.

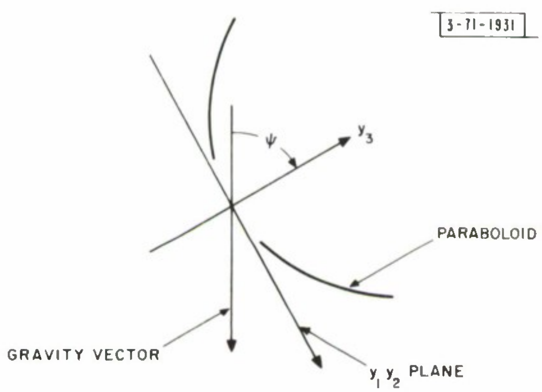



Fig. 7. Pointing angle.

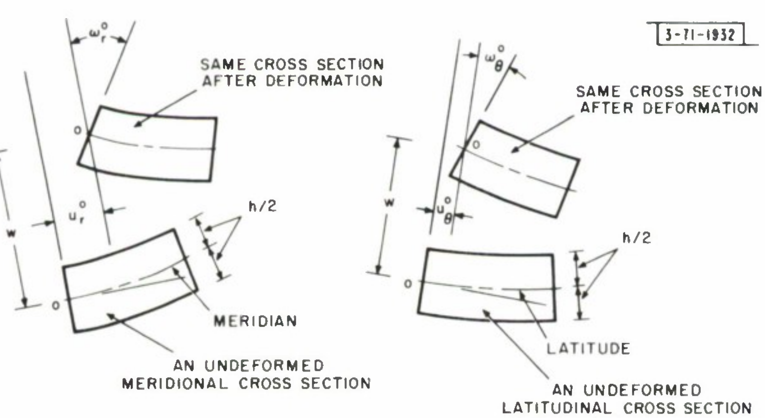



Fig. 8. Deformation of shell element.

Then the middle surface strains and rotations are given by the formulas listed below:

strain along meridian

$$\epsilon_r = \frac{1}{2f\sqrt{1+\gamma^2}}\frac{\partial u}{\partial r} - \frac{w}{2f(1+\gamma^2)^{3/2}} \tag{II-25}$$

strain along latitude

$$\epsilon_\Theta = \frac{1}{2f\gamma}\frac{\partial v}{\partial \Theta} + \frac{u}{2f\gamma\sqrt{1+\gamma^2}} - \frac{w}{2f\sqrt{1+\gamma^2}} \tag{II-26}$$

shearing strain

$$\epsilon_{r\Theta} = \frac{1}{2f\sqrt{1+\gamma^2}}\frac{\partial v}{\partial \gamma} - \frac{v}{2f\gamma\sqrt{1+\gamma^2}} + \frac{1}{2f\gamma}\frac{\partial u}{\partial \Theta} \tag{II-27}$$

rotation of $\overline{op}$ in meridional direction (see Fig. 8)

$$\omega_r = -\frac{u}{2f(1+\gamma^2)^{3/2}} - \frac{1}{2f\sqrt{1+\gamma^2}}\frac{\partial w}{\partial \gamma} \tag{II-28}$$

rotation of $\overline{op}$ in latitude direction (see Fig. 8)

$$\omega_\Theta = -\frac{v}{2f\sqrt{1+\gamma^2}} - \frac{1}{2f\gamma}\frac{\partial w}{\partial \Theta} \tag{II-29}$$

change in curvature along meridian

$$\kappa_r = \frac{1}{2f\sqrt{1+\gamma^2}}\frac{\partial \omega_r}{\partial \gamma} \tag{II-30}$$

change in curvature along latitude

$$\kappa_\Theta = \frac{\omega_r}{2f\gamma\sqrt{1+\gamma^2}} + \frac{1}{2f\gamma}\frac{\partial \omega_\Theta}{\partial \Theta} \tag{II-31}$$

twist

$$\kappa_{r\Theta} = \frac{1}{2}\left\{\frac{1}{2f\sqrt{1+\gamma^2}}\frac{\partial \omega_\Theta}{\partial \gamma} - \frac{\omega_\Theta}{2f\gamma\sqrt{1+\gamma^2}} + \frac{1}{2f\gamma}\frac{\partial \omega_r}{\partial \Theta}\right\} . \tag{II-32}$$

The stresses at point p in the shell are defined by means of Fig. 9. It is more convenient to define a system of force resultants and moment resultants acting at the middle surface of the shell which are equipollent to the stresses integrated over the thickness (Figs. 10 and 11). Let

$$N_r = \text{force resultant in meridional direction} = \int_{-h/2}^{h/2} \sigma_r \, d\zeta \tag{II-33}$$

$$N_\Theta = \text{force resultant in latitude direction} = \int_{-h/2}^{h/2} \sigma_\Theta \, d\zeta \tag{II-34}$$

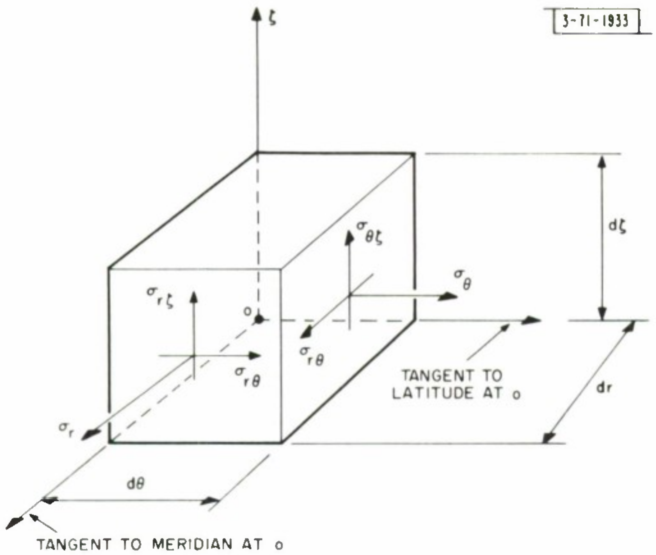
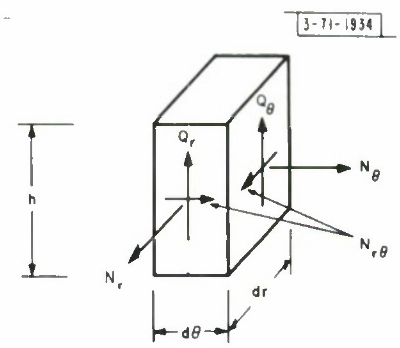



Fig. 9. Stresses in shell element.

Fig. 10. Stress resultants.



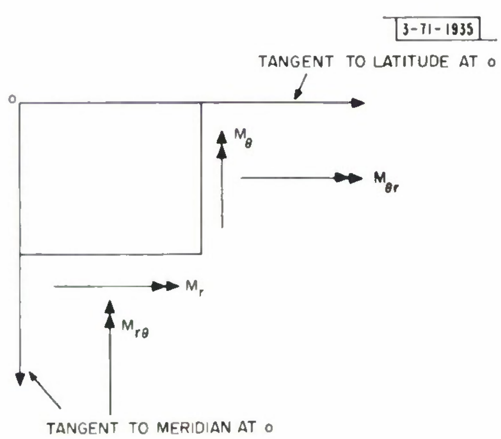




Fig. 11. Moment resultants. (The double-headed arrows represent moments. Use right-hand rule.)

$$N_{r\Theta} = N_{\Theta r} = \text{shear force resultant} = \int_{-h/2}^{h/2} \sigma_{r\Theta} d\zeta \tag{II-35}$$

$$Q_r = \text{transverse shear resultant on r face} = \int_{-h/2}^{h/2} \sigma_{r\zeta} d\zeta \tag{II-36}$$

$$Q_\Theta = \text{transverse shear resultant on } \Theta \text{ face} = \int_{-h/2}^{h/2} \sigma_{\Theta\zeta} d\zeta \tag{II-37}$$

$M_r$ = bending moment about tangent to latitude at o

$$= \int_{-h/2}^{h/2} \sigma_r \zeta d\zeta \tag{II-38}$$

$M_\Theta$ = bending moment about tangent to meridian at o

$$= \int_{-h/2}^{h/2} \sigma_\Theta \zeta d\zeta \tag{II-39}$$

$$M_{r\Theta} = M_{\Theta r} = \text{twisting moment} = \int_{-h/2}^{h/2} \sigma_{r\Theta} \zeta d\zeta \quad . \tag{II-40}$$

Then an element of volume bounded by surfaces r, r + dr, $\Theta$, $\Theta + d\Theta$, and $\pm h/2$ (see Fig. 3) satisfies the following equations of force and moment equilibrium.

$$\gamma \frac{\partial N_r}{\partial \gamma} + \sqrt{1+\gamma^2} \frac{\partial N_{r\Theta}}{\partial \Theta} + N_r - N_\Theta - \frac{\gamma}{1+\gamma^2} Q_r + 2f\gamma\sqrt{1+\gamma^2}\, p_r = 0 \tag{II-41}$$

$$\gamma \frac{\partial N_{r\Theta}}{\partial \gamma} + \sqrt{1+\gamma^2} \frac{\partial N_\Theta}{\partial \Theta} + 2N_{r\Theta} - \gamma Q_\Theta + 2f\gamma\sqrt{1+\gamma^2}\, p_\Theta = 0 \tag{II-42}$$

$$\gamma \frac{\partial Q_r}{\partial \gamma} + \sqrt{1+\gamma^2} \frac{\partial Q_\Theta}{\partial \Theta} + \frac{\gamma}{1+\gamma^2} N_r + \gamma N_\Theta + Q_r + 2f\gamma\sqrt{1+\gamma^2}\, p_n = 0 \tag{II-43}$$

$$\gamma \frac{\partial M_r}{\partial \gamma} + \sqrt{1+\gamma^2} \frac{\partial M_{r\Theta}}{\partial \Theta} + M_r - M_\Theta - 2f\gamma\sqrt{1+\gamma^2}\, Q_r = 0 \tag{II-44}$$

$$\gamma \frac{\partial M_{r\Theta}}{\partial \gamma} + \sqrt{1+\gamma^2} \frac{\partial M_\Theta}{\partial \Theta} + 2M_{r\Theta} - 2f\gamma\sqrt{1+\gamma^2}\, Q_\Theta = 0 \tag{II-45}$$

where $p_r$ and $p_\Theta$ are the components of the applied surface loads along the tangent to meridian and to a latitude, respectively; $p_n$ is the component along the normal to the middle surface. For a shell subjected to gravity load with an arbitrary pointing angle $\psi$,

$$p_r = \frac{\rho}{\sqrt{1+\gamma^2}} [\sin\Theta \sin\psi - \gamma \cos\psi] \tag{II-46}$$

$$p_\Theta = \rho \cos\Theta \sin\psi \tag{II-47}$$

and

$$p_n = -\frac{\rho}{\sqrt{1+\gamma^2}} [\gamma \sin\Theta \sin\psi - \cos\psi] \tag{II-48}$$

where $\rho$ is the surface weight density of the shell (lb/in.$^2$) and is related to the volume weight density $\rho_o$ by

$$\rho = \int_{-h/2}^{h/2} \rho_o d\zeta \quad . \tag{II-49}$$

For a homogeneous shell, we have $\rho = \rho_o h$.

For an isotropic shell, the stress resultants and the moment resultants are related to the shell middle-surface strains and curvature changes as follows:

$$N_r = C(\epsilon_r + \nu_m \epsilon_\theta - \Delta\overline{T}) \tag{II-50}$$

$$N_\theta = C(\epsilon_\theta + \nu_m \epsilon_r - \Delta\overline{T}) \tag{II-51}$$

$$N_{r\theta} = C(1 - \nu_m)\, \epsilon_{r\theta} \tag{II-52}$$

$$M_r = D(\kappa_r + \nu_b \kappa_\theta - \Delta\widetilde{T}) \tag{II-53}$$

$$M_\theta = D(\kappa_\theta + \nu_b \kappa_r - \Delta\widetilde{T}) \tag{II-54}$$

$$M_{r\theta} = D(1 - \nu_b)\, \kappa_{r\theta} \quad . \tag{II-55}$$

The extensional stiffness C, the bending stiffness D, the effective stretching and bending Poisson's ratios $\nu_m$ and $\nu_b$ are given in terms of the Young's modulus E and Poisson's ratio $\nu$ by

$$C = \int_{-h/2}^{h/2} \frac{E}{1 - \nu^2}\, d\zeta \tag{II-56}$$

$$\nu_m = \frac{1}{C} \int_{-h/2}^{h/2} \frac{\nu E}{1 - \nu^2}\, d\zeta \tag{II-57}$$

$$D = \int_{-h/2}^{h/2} \frac{E}{1 - \nu^2}\, \zeta^2 d\zeta \tag{II-58}$$

$$\nu_b = \frac{1}{D} \int_{-h/2}^{h/2} \frac{\nu E}{1 - \nu^2}\, \zeta^2 d\zeta \quad . \tag{II-59}$$

The membrane and bending thermal strains $\Delta\overline{T}$ and $\Delta\widetilde{T}$, are given in terms of E, $\nu$, the change in temperature $\Delta T$, and the coefficient of thermal expansion of the shell $\alpha$ by

$$\Delta\overline{T} = \frac{1}{C} \int_{-h/2}^{h/2} \frac{\alpha E \Delta T}{1 - \nu}\, d\zeta \tag{II-60}$$

$$\Delta\widetilde{T} = \frac{1}{D} \int_{-h/2}^{h/2} \frac{\alpha E \Delta T}{1 - \nu}\, \zeta d\zeta \quad . \tag{II-61}$$

If the shell is homogeneous in the thickness direction, then

$$\nu_m = \nu_b = \nu \tag{II-62}$$

$$C = \frac{Eh}{1 - \nu^2} \qquad \text{(II-63)}$$

$$D = \frac{Eh^3}{12(1 - \nu^2)} \qquad \text{(II-64)}$$

$$\Delta\overline{T} = \frac{\alpha(1 + \nu)}{h} \int_{-h/2}^{h/2} \Delta T d\zeta \qquad \text{(II-65)}$$

$$\Delta\widetilde{T} = \frac{12(1 + \nu)\,\alpha}{h} \int_{-h/2}^{h/2} \Delta T \zeta d\zeta \quad . \qquad \text{(II-66)}$$

We shall have occasion in the subsequent development to refer to a parameter defined by

$$\lambda^2 = \frac{r_2^2}{\sqrt{R^2 AD}} \qquad \text{(II-67)}$$

where R is the representative magnitude of the principal radii of curvature. For a paraboloidal shell of revolution, we have R = 2f. If, in addition, the shell is isotropic and homogeneous, we have

$$\lambda = \frac{r_2}{\sqrt{2fh}} \sqrt[4]{12(1 - \nu^2)} = O\left(\frac{r_2}{\sqrt{2fh}}\right) \quad . \qquad \text{(II-68)}$$

Equations (II-25) through (II-32), (II-41) through (II-45), and (II-50) through (II-55) form a set of nineteen equations for the nineteen structural parameters ($N_r$, $N_\Theta$, $N_{r\Theta}$, $Q_r$, $Q_\Theta$, $M_r$, $M_\Theta$, $M_{r\Theta}$, $\epsilon_r$, $\epsilon_\Theta$, $\epsilon_{r\Theta}$, $\omega_r$, $\omega_\Theta$, $\kappa_r$, $\kappa_\Theta$, $\kappa_{r\Theta}$, u, v, and w). They are the shell equations for a paraboloidal shell of revolution.

The components of stress at a point o are related to the stress and moment resultants by the following formulas.

$$\sigma_r = \frac{N_r}{h} + \frac{12M_r}{h^3}\zeta \qquad \text{(II-69)}$$

$$\sigma_\Theta = \frac{N_\Theta}{h} + \frac{12M_\Theta}{h^3}\zeta \qquad \text{(II-70)}$$

$$\sigma_{r\Theta} = \frac{N_{r\Theta}}{h} + \frac{12M_{r\Theta}}{h^3}\zeta \qquad \text{(II-71)}$$

$$\sigma_{r\zeta} = \frac{3Q_r}{2h}\left(1 - \frac{4\zeta^2}{h^2}\right) \qquad \text{(II-72)}$$

$$\sigma_{\Theta\zeta} = \frac{3Q_\Theta}{2h}\left(1 - \frac{4\zeta^2}{h^2}\right) \quad . \qquad \text{(II-73)}$$

C. EDGES OF SHELL

Along a $\gamma$ = constant edge, we may prescribe an appropriate combination of:

(1) Any one of the four quantities

(a) Normal displacement $w$

(b) Axial displacement $u_v$

$$u_v = w \cos\varphi + u \sin\varphi \tag{II-74}$$

(c) Effective transverse shear $Q_r + (1/2f\gamma)(\partial M_{r\theta}/\partial\theta)$

(d) Effective axial resultant $V$

$$V = \left(Q_r + \frac{1}{2f\gamma}\frac{\partial M_{r\theta}}{\partial\theta}\right)\cos\varphi + N_r \sin\varphi \quad . \tag{II-75}$$

(2) Any one of the four quantities

(a) Meridional displacement $u$

(b) Radial displacement $u_h$

$$u_h = -w \sin\varphi + u \cos\varphi \tag{II-76}$$

(c) Meridional stress resultant $N_r$

(d) Effective radial resultant $H$

$$H = -\left(Q_r + \frac{1}{2f\gamma}\frac{\partial M_{r\theta}}{\partial\theta}\right)\sin\varphi + N_r \cos\varphi \tag{II-77}$$

(3) Either the circumferential displacement $v$ or the effective shear resultant $N_{r\theta} + (M_{r\theta}/R_\theta)$, and

(4) Either the moment $M_r$ or the rotation $\omega_r$, where

$$\cos\varphi = \frac{1}{\sqrt{1+\gamma^2}} \tag{II-78}$$

$$\sin\varphi = \frac{\gamma}{\sqrt{1+\gamma^2}} \quad . \tag{II-79}$$

A combination of prescribed conditions is appropriate if the structure is in global static equilibrium under these conditions.

The standard idealized edges supports are merely some special combinations of the above general set. For a simply supported edge, we have

$$w = u = v = M_r = 0 \quad . \tag{II-80}$$

For a clamped edge, we have

$$w = \omega_r = u = v = 0 \quad . \tag{II-81}$$

For a free edge, we have

$$N_r = N_{r\theta} + \frac{M_{r\theta}}{R_\theta} = M_r = Q_r + \frac{1}{2f\gamma}\frac{\partial M_{r\theta}}{\partial\theta} = 0 \quad . \tag{II-82}$$

If the shell is closed at the apex, we require that the stresses and displacements be finite at $\gamma = 0$.

## D. SOLUTION TO SHELL EQUATIONS FOR GRAVITY LOAD

For a shell under gravity load and without edge loads and temperature gradients, the solution to our shell equations must take the following form.[4]

$$(N_r, N_\theta, Q_r, M_r, M_\theta, u, w, \omega_r) = (N_r^s, N_\theta^s, Q_r^s, M_r^s, M_\theta^s, u^s, w^s, \omega_r^s) \cos\psi$$

$$+ (N_r^a, N_\theta^a, Q_r^a, M_r^a, M_\theta^a, u^a, w^a, \omega_r^a) \sin\theta \sin\psi \tag{II-83}$$

$$(N_{r\theta}, M_{r\theta}, Q_\theta, v) = (N_{r\theta}^a, M_{r\theta}^a, Q_\theta^a, v^a) \cos\theta \sin\psi \quad . \tag{II-84}$$

In the subsequent development, we will not be concerned with the other seven structural quantities which can be obtained by appropriate combinations of those previously given. It should be said at this point that, although the form of the solution to our shell equations has been obtained with relative ease, the exact solution to these same equations does not seem likely. The computer programs presented herein represent several approaches to an approximate solution. These different approaches will be described under the appropriate programs and more thoroughly discussed in Refs. 1-5. In the remaining portion of this section, we shall discuss some general features of the output of these programs in connection with the shell behavior from the designer's point of view.

The superscripted quantities appearing in Eqs. (II-83) and (II-84) are independent of $\psi$ and $\theta$. Their dependence on $\gamma$ is generally complicated. It is clear from the same equations that these are the key quantities to our problem. Once they are determined, the physical quantities appearing on the left-hand sides of Eqs. (II-83) and (II-84) can be obtained for any value of $\psi$ and $\theta$ by straightforward calculations. The computer programs discussed herein are designed mainly to calculate these superscripted quantities for a given set of values of $\gamma$ which will be referred to as the $\gamma$-tabular points. However, since $\psi$ is really a load parameter, its effect on the stresses and deformations of the shell is generally of prime interest to the designers of larger antennas. Therefore, the programs for gravity load give generally the following quantities as its output.

$$(N_r^o, N_\theta^o, \ldots, w^o, \omega_r^o) = (N_r^s, N_\theta^s, \ldots, w^s, \omega_r^s) \cos\psi$$

$$+ (N_r^a, N_\theta^a, \ldots, w^a, \omega_r^a) \sin\psi \tag{II-85}$$

$$(N_{r\theta}^o, M_{r\theta}^o, Q_\theta^o, v^o) = (N_{r\theta}^a, \ldots, v^a) \sin\psi \quad . \tag{II-86}$$

The various physical quantities themselves will also be calculated upon request. To this end, we must prescribe, in addition to the $\gamma$-tabular points, a set of values for $\theta$ which will be referred to as the $\theta$-tabular points. Together with the former, they form a net of points each describing a particular point on the middle surface of the shell. The programs calculate the stress and displacement of the shell for this net of tabular points. In all cases, the unit for length is inch and the unit for force is pound. While the shell is closed in the circumferential direction so that the range of $\theta$ is $(0, 2\pi)$, it is clearly sufficient to calculate for any particular latitude (i.e., for any fixed value of $\gamma$) the values of the physical quantities for $o \leqslant \theta \leqslant \pi$. This rather trivial observation may save a great deal of computing time if a large number of runs is desired.

It is often desirable to present numerieal results in a form whieh is independent of the various geometrical and material parameters assoeiated with the shell. In general, this is not possible in shell analysis. Thus, we can only hope to find normalizing factors which yield dimensionless quantities of the same order of magnitude for shells with different geometry and different materials. To this end, we let

$$(u^*, v^*, w^*) = \frac{C(1-\nu^2)}{4f^2\rho}(u, v, w) \tag{II-87}$$

$$(N_r^*, N_\theta^*, N_{r\theta}^*) = \frac{1}{2f\rho}(N_r, N_\theta, N_{r\theta}) \tag{II-88}$$

$$\left(M_r^*, M_\theta^*, \frac{M_{r\theta}^*}{mk_o}\right) = \frac{(mk_o)^2}{4f^2\rho}(M_r, M_\theta, M_{r\theta}) \tag{II-89}$$

$$\left(Q_r^*, \frac{Q_\theta^*}{mk_o}\right) = \frac{mk_o}{2f\rho}(Q_r, Q_\theta) \tag{II-90}$$

$$\omega_r^* = \frac{C(1-\nu^2)}{2f\rho(mk_o)}\omega_r \tag{II-91}$$

$$k_o^4 = \frac{4f^2C}{D} \tag{II-92}$$

$$m^4 = (1-\nu^2) \quad . \tag{II-93}$$

Although the starred quantities are not invariant with respeet to the various geometrical and material properties of the shell (for instanee, they obviously depend on Poisson's ratio $\nu$), their magnitude does not vary appreeiably even with appreeiable changes in the relevant geometrical and material parameters. Both the starred (normalized) quantities and the unstarred (unnormalized) quantities may be obtained from our eomputer programs.

It ean be seen from an examination of the solution that a portion of the displacements corresponds to a rigid body translation and/or rigid body rotation about the $y^1$ axis. For example, in Group Report 71G-1-II (Ref. 2), the constant $C_2$ in Eq. (6.4.21) for w (membrane analysis) is seen to represent a translation of the shell parallel to the $y^3$ axis. Sinee this is a rigid body motion, $C_2$ does not appear in any of the equations for the force resultants. Similarly, in the asymptotic analysis, the constant $C_2$ [Eqs. (8.9.1) and (8.9.2), Group Report 71G-1, Part IV (Ref. 4)] denotes a rigid body translation parallel to the $y^3$ axis for symmetrie loads. For antisymmetrie loads, the eonstants $C_3$ and $C_4$ [Eqs. (8.10.1) to (8.10.3), Group Report 71G-1, Part IV (Ref. 4)] represent a rigid body rotation about the $y^1$ axis and a rigid body translation parallel to the $y^2$ axis, respeetively. The rigid body displaeements do not affeet the shape of the paraboloidal surface and henee can be interpreted as a change in the position of the focal point. We will denote the defleetions due to rigid body translations and/or rotation by $\hat{u}$, $\hat{v}$, and $\hat{w}$. Then, the actual flexibility, i.e., distortions, will be given by

$$u_{flex} = u - \hat{u} \tag{II-94}$$

$$v_{flex} = v - \hat{v} \tag{II-95}$$

$$w_{flex} = w - \hat{w} \quad . \tag{II-96}$$

From the previous discussion, it is clear that these quantities should be of considerable interest to the designers of large antennas. Our computer program will display these as part of the output.

Thus far, it has been assumed that the shell has its prescribed shape before the introduction of the applied loads. Once loaded, the distortion of the shell is taken with respect to this prescribed (paraboloidal) shape. The antenna designer may also be interested in the distortions of the shell relative to its shape when the axis of the shell coincides with gravity vector. This is usually the attitude of the shell during erection and the designer may elect to construct the shell to a prescribed shape while it is in this attitude. Thus, we are often interested in

$$\tilde{w} = w^{s}(\cos\psi - 1) + w^{a} \sin\Theta \sin\psi \qquad \text{(II-97)}$$

$$\tilde{u} = u^{s}(\cos\psi - 1) + u^{a} \sin\Theta \sin\psi \qquad \text{(II-98)}$$

$$\tilde{v} = v^{a} \cos\Theta \sin\psi \qquad \text{(II-99)}$$

where the tilde quantities are the components of displacement relative to the face-up (i.e., $\psi = 0$) position. The corresponding quantities $\tilde{u}_{flex}$, $\tilde{v}_{flex}$, $\tilde{w}_{flex}$, $\tilde{u}^{o}$, $\tilde{v}^{o}$, $\tilde{w}^{o}$, $\tilde{u}^{o}{}_{flex}$, $\tilde{v}^{o}{}_{flex}$, and $\tilde{w}^{o}{}_{flex}$ are defined in the obvious way.

## III. MEMBRANE ANALYSIS OF GRAVITY LOAD

### A. MEMBRANE SOLUTION

Under conducive circumstances, the interior of the shell behaves like a membrane. Away from the edges of the structure, a good approximation of the behavior of the shell can be obtained by the so-called membrane (or momentless) theory of shell. Such a theory assumes that the shell has no bending stiffness, and quantities associated with the bending action of the shell vanish identically, i.e.,

$$M_r = M_\theta = M_{r\theta} = Q_r = Q_\theta = 0 \quad . \tag{III-1}$$

Consequently, Eqs. (II-41) to (II-43) become

$$\gamma \frac{\partial N_r}{\partial \gamma} + \sqrt{1+\gamma^2}\, \frac{\partial N_{r\theta}}{\partial \theta} + N_r - N_\theta + 2f\gamma \sqrt{1+\gamma^2}\, p_r = 0 \tag{III-2}$$

$$\gamma \frac{\partial N_{r\theta}}{\partial \gamma} + \sqrt{1+\gamma^2}\, \frac{\partial N_\theta}{\partial \theta} + 2N_{r\theta} + 2f\gamma \sqrt{1+\gamma^2}\, p_\theta = 0 \tag{III-3}$$

$$\frac{N_r}{1+\gamma^2} + N_\theta + 2f \sqrt{1+\gamma^2}\, p_n = 0 \tag{III-4}$$

and Eqs. (II-44) and (II-45) are identically satisfied.

Equations (III-2) to (III-4) contain only three unknowns and can therefore be solved for the stress resultants $N_r$, $N_\theta$, and $N_{r\theta}$. In other words, the membrane problem is statically determinate. The solution for $N_r$, $N_\theta$, and $N_{r\theta}$ can then be inserted in the left-hand side of Eqs. (II-25) to (II-27) by way of Eqs. (II-50) to (II-52) with the temperature terms omitted. The three strain-displacement relations also contain only three unknowns and can therefore be solved for u, v, and w.

Note that associated with the momentless assumption is a reduction of the order of our system of differential (shell) equations from eight to four. Thus, only half the conditions at each edge (Sec. II-C) can be satisfied. For a free edge, the last two conditions [Eq. (II-82)] are satisfied identically; therefore, we are left with the necessary two boundary conditions. In the case of a supported edge, it has been shown[5] that the appropriate boundary conditions are

$$u = 0 \tag{III-5}$$

and

$$v = 0 \quad . \tag{III-6}$$

Since the shell can not be acted upon by transverse forces or bending moments, it seems reasonable that it may not be constrained in the normal direction.

The exact analytical solution for the membrane stress resultants as well as the middle-surface displacement components in accordance with the momentless theory have been obtained and tabulated in Ref. 2. It can be seen from these analytical solutions that the corresponding normalizing factors given in Sec. II-D are nearly the true scale factors for these quantities. For a fixed value of $\nu$, one set of normalized results generated from our computer program is valid for the entire class of geometrically similar shells, i.e., shells with the same $\gamma_1$ and $\gamma_2$.

The analytic expressions for the various physical quantities determined by the momentless theory of shell are formed by complicated combinations of elementary functions. To evaluate

them for a large number of tabular points even by a desk calculator is a formidable task. A computer program is written to make this information easily accessible to the designers of large antennas and to others who might be interested. The program generates the tabular points, computes the constants of integration, and calculates the stress resultants and displacements of the middle surface.

## B. DIGITAL COMPUTER PROGRAM

The general scheme of the computer program for a membrane analysis is outlined in Fig. 12, the master flow chart. At the end of a complete run, the program returns to step (1). When it

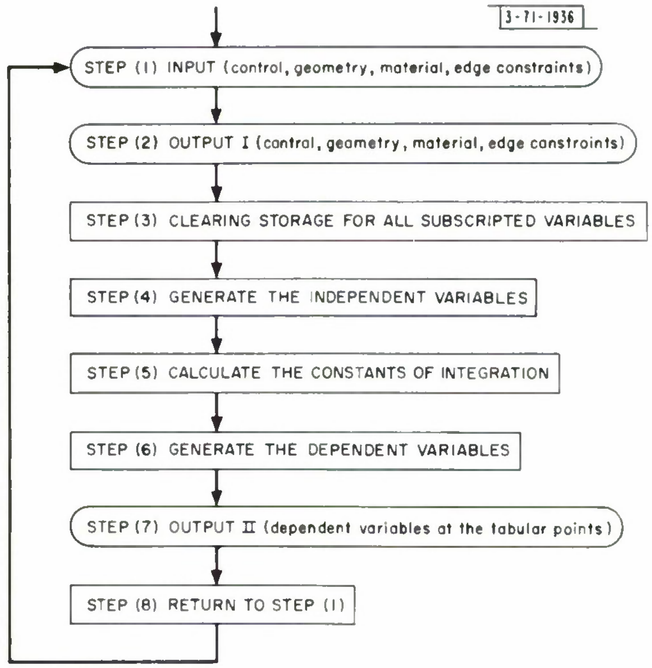



Fig. 12. Master flow chart.

fails to locate additional input, the program exits automatically. This is a special feature of the Lincoln Laboratory Express Runs. Slight modifications of the ending will be necessary if the program is to be run differently or if some indication of a successful run is desired. The same Express Runs also require that the input be prestored on machine tape A2 and that the output be written on A3. In complying with this restriction, the program reads all its input from A2 and writes all its output on A3. For the purpose of the Express Runs, the object deck must be preceded by two cards each with an asterisk in column one. The first of these is an identification card while the second contains the work XEQ occupying columns seven, eight, and nine (Sample Card 1). Following the object deck is another card with an asterisk in column one and the word DATA occupying columns seven, eight, nine, and ten (Sample Card 2). Then comes the input to the program to be prestored on A2. The program listing will be presented in Table I.

Sample Card 1 XEQ

* XEQ

Sample Card 2 DATA

* DATA

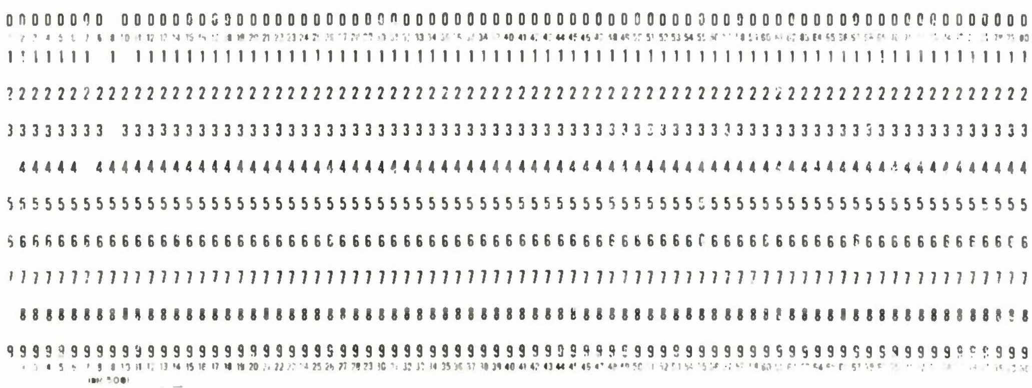


TABLE I

PROGRAM LISTING

```
*      SHEA-WAN       MEMBRANE SHELL WITH GRAVITY LOAD                9/3/64
*      XEQ
C      WAN, FRED      MEMBRANE SHELL WITH GRAVITY LOAD                9/3/64
       DIMENSION GAMMA(150),THETA(10),ENR(150,10),ENT(150,10),ENRT(150,10
      1),U(150,10),V(150,10),W(150,10),EN1(150),EN2(150),U1(150),W1(150),
      2R(150),A(150),B(150),C(150),G(150),P(150),Q(150),D(3,3),Y(3),Z(3),
      3GAMMR(150),S(150),FMT1(12),FMT2(12)
       DIMENSION  UFLEXA(150),VFLEXA(150),WFLEXA(150),UFLEXS(150),WFLEXS
      1(150),UFLEX(150,10),VFLEX(150,10),WFLEX(150,10)
       COMMON ENR,ENT,ENRT,V,W,U
       EQUIVALENCE (GAMMA(1),R(1)),(GAMMR(1),S(1))
C
C - - (1) INPUT
C
   10 READ INPUT TAPE 2,101,M,N,KE,NORM,KD,KP
      READ INPUT TAPE 2,105,R1,R2,R3,R4
      READ INPUT TAPE 2,105,F,PR,PSI
      X1=1.+PR
      X6=1.-PR
      X7=X1*X6
      IF(KP-1)320,321,322
  320 READ INPUT TAPE 2,102,RHO,H,E
      RHO=RHO*H
      CC=E*H/X7
      GO TO 323
  321 READ INPUT TAPE 2,102,RHO,CC
      GO TO 323
  322 READ INPUT TAPE 2,102,RHS,RHC,T,H,E
      RHO=2.*T*RHS+(H-T)*RHC
      CC=2.*T*E/X7
  323 IF(N)7,7,8
    8 READ INPUT TAPE 2,105,THETA3,THETA4
      READ INPUT TAPE 2,205,FMT1,FMT2
C
C - - (2) OUTPUT I
C
    7 WRITE OUTPUT TAPE 3,107
      IF(KP-1)67,68,69
   67 WRITE OUTPUT TAPE 3,160
      GO TO 73
   68 WRITE OUTPUT TAPE 3,161
      GO TO 73
   69 WRITE OUTPUT TAPE 3,162
   73 IF(NORM)74,74,75
   74 WRITE OUTPUT TAPE 3,139
      GO TO 302
   75 WRITE OUTPUT TAPE 3,140
  302 IF(KE-1)93,92,50
   93 WRITE OUTPUT TAPE 3,115
      GO TO 91
   92 WRITE OUTPUT TAPE 3,116
      GO TO 91
   50 WRITE OUTPUT TAPE 3,118
   91 IF(KP-1)76,77,78
   76 WRITE OUTPUT TAPE 3,112
      WRITE OUTPUT TAPE 3,111,RHO,E,H,F,PR
      GO TO 79
   77 WRITE OUTPUT TAPE 3,163
```

TABLE I (Continued)

```
      WRITE OUTPUT TAPE 3,111,CC,RHO,F,PR
      GO TO 79
   78 WRITE OUTPUT TAPE 3,164
      WRITE OUTPUT TAPE 3,111,E,H,T,RHS
      WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,165
      WRITE OUTPUT TAPE 3,111,RHC,RHO,F,PR
   79 WRITE OUTPUT TAPE 3,114
      PI =3.14159265
      RAD=57.2957795
      PSIDEG=PSI*PI*RAD
      WRITE OUTPUT TAPE 3,113
      WRITE OUTPUT TAPE 3,108,R1,R2,R3,R4,PSIDEG
      WRITE OUTPUT TAPE 3,114
      IF(N)1,1,9
    9 T3DEG=THETA3*PI*RAD
      T4DEG=THETA4*PI*RAD
      WRITE OUTPUT TAPE 3,117
      WRITE OUTPUT TAPE 3,119,T3DEG,T4DEG
    1 WRITE OUTPUT TAPE 3,114
C
C - - (3) CLEARING STORAGE FOR SUBSCRIPTED VARIABLES
C
      DO 51 I=1,150
      EN1(I)=0.
      EN2(I)=0.
      U1(I)=0.
      W1(I)=0.
      A(I)=0.
      B(I)=0.
      C(I)=0.
      G(I)=0.
      P(I)=0.
      Q(I)=0.
      GAMMR(I)=0.
   51 GAMMA(I)=0.
      DO 52 J=1,10
      THETA(J)=0.
      DO 52 I=1,150
      U(I,J)=0.
      W(I,J)=0.
      V(I,J)=0.
      ENR(I,J)=0.
      ENT(I,J)=0.
   52 ENRT(I,J)=0.
C
C - - (4) GENERATE INDEPENDENT VARIABLES
C
      F2=2.*F
      UB=.00000005
      TWOPI=2.*PI
      R1=R1/F2
      R2=R2/F2
      R3=R3/F2
      R4=R4/F2
      M1=M-1
      EM1=M1
      DELTR=R4-R3
```

TABLE I (Continued)

```
      DELTR=DELTR/EM1
      GAMMA(1)=R3
      DO 30 I=1,M1
      I1=I+1
      EI=I
   30 GAMMA(I1)=R3+DELTR*EI
      IF(NORM)630,630,632
  630 DO 631 I=1,M
  631 GAMMR(I)=GAMMA(I)
      GO TO 634
  632 DO 633 I=1,M
  633 GAMMR(I)=GAMMA(I)*F2
  634 IF(N)5,5,40
   40 THETA3=THETA3*PI
      THETA4=THETA4*PI
      N1=N-1
      AN1=N1
      THETA5=THETA4-THETA3
      DELTT=THETA5/AN1
      THETA(1)=THETA3
      DO 2 I=1,N1
      I1=I+1
    2 THETA(I1)=THETA(I)+DELTT
      IF(THETA4-TWOPI) 5,6,6
    6 THETA(N)=THETA3
C
C - - (5) CALCULATE THE CONSTANTS OF INTEGRATION
C
    5 CS=F2*RHO
      CD=F2**2*RHO/(X7*CC)
      PHI=PI*PSI
      CPHI=COSF(PHI)
      SPHI=SINF(PHI)
      IF(PSI)38,31,32
   38 WRITE OUTPUT TAPE 3,143
      GO TO 10
   32 CSS=CS*CPHI/3.
      CDS=CD*CPHI/3.
      CSA=2.*CS*SPHI
      CDA=CD*SPHI
   31 X2=.5*(3.-PR)
      X3=.25
      X4=-2.*(1.-PR)
      X5=PR
      IF(KE-1)95,96,98
   98 A1=-1.
      C1=-1./3.
      C2=-1./15.
      ARG1=1.+R2**2
      ARG1=SQRTF(ARG1)
      X0=-(1.+PR)*A1
      A3=-ARG1*(A1*R2+X0/R2)-X0*R2*LOGF(1.+ARG1)+(X0-X1)*R2*LOGF(R2)-X2*
     1R2**3-X3*R2**5+X1/R2
      A3=A3/R2
      GO TO 94
   96 A1=1.+R1**2
      A1=SQRTF(A1)
      A1=-A1**3
```

TABLE I (Continued)

```
      ARG1=1.+R2**2
      ARG1=SQRTF(ARG1)
      X0=-(1.+PR)*A1
      A3=-ARG1*(A1*R2+X0/R2)-X0*R2*LOGF(1.+ARG1)+(X0-X1)*R2*LOGF(R2)-X2*
     1R2**3-X3*R2**5+X1/R2
      A3=A3/R2
      ARG1=1.+R1**2
      ARG1=SQRTF(ARG1)
      C1=-ARG1**3/3.
      C2=-(.5*C1*R1**2+ARG1**5/15.)
   94 ARG1=SQRTF(1.+R2**2)
      ARG2=ARG1-1.
      ARG3=ARG2/R2
      H1=ARG1*(R2  *.5-X1/R2  )
      H2=X6*.5*R2  *LOGF(ARG3)+X1*ARG1*(1.-.5*   R2  **2)/R2  **3
      H3=R2**5-(40.+29.*PR    )*R2**3+X6*2.*R2*LOGF(R2)+X1*(4.-20.*R2**2
     1)/R2**3
      H3=H3/60.
      G1=-X1*(ARG1+LOGF(ARG3))+ARG1**3/6.
      G2=.25*X6*(R2  **2*LOGF(ARG3)+1.+ARG1)-.5*X1*ARG1**3/R2  **2
      G3=R2  **6-1.5*(40.+29.*PR    )*R2  **4-3.*X6*R2  **2+LOGF(R2  )*(
     16.*X6*R2  **2-X1*120.)-X1* 12./R2  **2
      G3=G3/360.
      TERM1=-H2+2.*X1*ARG1/R2**3
      TERM2=H1+   X1*ARG1/R2
      TERM3=-H3+2.*X1*(1.-4.*R2**2)*ARG1**4/(15.*R2**3)
      C3=(TERM3+C2*TERM1-TERM2*C1)/R2
      C4=-(.5*C3*R2**2+C2*G2+C1*G1+G3)
      GO TO 54
   95 ARG11=SQRTF(1.+R1**2)
      ARG12=SQRTF(1.+R2**2)
      ARG3=(1.+ARG11)/R1
      ARG4=(1.+ARG12)/R2
      T1=R1*ARG11-X1*(R1*LOGF(ARG3)+ARG11/R1)
      T2=R2*ARG12-X1*(R2*LOGF(ARG4)+ARG12/R2)
      T3=-X3*R1**5-X2*R1**3-(R1*LOGF(R1)-1./R1)*X1
      T4=-X3*R2**5-X2*R2**3-(R2*LOGF(R2)-1./R2)*X1
      E1=R2*T3-T4*R1
      E2=T3*T2-T4*T1
      DENOM=T1*R2-T2*R1
      A1=E1/DENOM
      A3=-E2/DENOM
      ARG21=ARG11-1.
      ARG22=ARG12-1.
      ARG31=ARG21/R1
      ARG32=ARG22/R2
      H11=ARG11*(R1*.5-X1/R1)
      H12=ARG12*(R2*.5-X1/R2)
      H22=X6*.5*R2*LOGF(ARG32)+X1*ARG12*(1.-.5*R2**2)/R2**3
      H21=X6*.5*R1*LOGF(ARG31)+X1*ARG11*(1.-.5*R1**2)/R1**3
      H31=R1**5-(40.+29.*PR    )*R1**3+X6*2.*R1*LOGF(R1)+X1*(4.-20.*R1**
     12)/R1**3
      H32=R2**5-(40.+29.*PR    )*R2**3+X6*2.*R2*LOGF(R2)+X1*(4.-20.*R2**
     12)/R2**3
      H31=H31/60.
      H32=H32/60.
      G11=-X1*(ARG11+LOGF(ARG31))+ARG11**3/6.
      G12=-X1*(ARG12+LOGF(ARG32))+ARG12**3/6.
```

TABLE I (Continued)

```
      G21=.25*X6*(R1**2*LOGF(ARG31)+1.+ARG11)-.5*X1*ARG11**3/R1**2
      G22=.25*X6*(R2**2*LOGF(ARG32)+1.+ARG12)-.5*X1*ARG12**3/R2**2
      G31=R1**6-1.5*(40.+29.*PR     )*R1**4-3.*X6*R1**2+LOGF(R1)*(6.*X6*R
     11**2-X1*120.)-X1*12./R1**2
      G32=R2**6-1.5*(40.+29.*PR     )*R2**4-3.*X6*R2**2+LOGF(R2)*(6.*X6*R
     12**2-X1*120.)-X1*12./R2**2
      G31=G31/360.
      G32=G32/360.
      D(1,1)=G11-G12
      D(1,2)=G21-G22
      D(1,3)=.5*(R1**2-R2**2)
      D(2,1)=H11+X1*ARG11/R1
      D(2,2)=H21-2.*X1*ARG11/R1**3
      D(2,3)=R1
      D(3,1)=H12+X1*ARG12/R2
      D(3,2)=H22-2.*X1*ARG12/R2**3
      D(3,3)=R2
      Z(1)=G32-G31
      Z(2)=-H31+2.*X1*ARG11**4*(1.-4.*R1**2)/(15.*R1**3)
      Z(3)=-H32+2.*X1*ARG12**4*(1.-4.*R2**2)/(15.*R2**3)
      DET=1.
      L=XSIMEQF(3,3,1,D,Z,DET,Y)
      GO TO (70,71,72),L
   72 WRITE OUTPUT TAPE 3,103
      GO TO 10
   71 WRITE OUTPUT TAPE 3,104
      GO TO 10
   70 C1=D(1,1)
      C2=D(2,1)
      C3=D(3,1)
      C4=-(G31+C1*G11+C2*G21+C3*.5*R1**2)
      X0=-X1*A1
C
C - - (6) GENERATE THE DEPENDENT VARIABLES
C
   54 IF(PSI)38,35,39
   39 DO 4 I=1,M
      ARG3=1.+GAMMA(I)**2
      ARG4=SQRTF(ARG3)
      ARG1=ARG3-1.
      ARG2=ARG1/GAMMA(I)
      BEBA=(.5*C1*ARG4/R(I))+(C2*ARG4/R(I)**3)+(ARG3**3/(15.*R(I)**3))
      BEBA=-BEBA
      BEBC=(-.5*C1/R(I))+(C2/R(I)**3) +(ARG4**3*(1.-4.*R(I)**2)/(15.*R(I
     1)**3))
      BEBB=.5*R(I)+((C2/R(I)**3)+(.5*C1/R(I))+ARG4*ARG3**2/(15.*R(I)**3)
     1)/ARG4
      A(I)=CSA*BEBA
      B(I)=CSA*BEBB
      C(I)=CSA*BEBC
      ARG1=SQRTF(1.+R(I)**2)
      ARG2=ARG1-1.
      ARG3=ARG2/R(I)
      G1=-X1*(ARG1+LOGF(ARG3))+ARG1**3/6.
      G2=.25*X6*(R(I)**2*LOGF(ARG3)+1.+ARG1)-.5*X1*ARG1**3/R(I)**2
      G3=R(I)**6-1.5*(40.+29.*PR     )*R(I)**4-3.*X6*R(I)**2+LOGF(R(I))*(
     16.*X6*R(I)**2-X1*120.)-X1* 12./R(I)**2
      G3=G3/360.
```

TABLE I (Continued)

```
      VFLEXA(I)=2.*CDA*(C1*G1+C2*G2+G3)
      G(I)=2.*(C4+.5*C3*R(I)**2+C2*G2+C1*G1+G3)*CDA
      H1=ARG1*(R(I)*.5-X1/R(I))
      H2=X6*.5*R(I)*LOGF(ARG3)+X1*ARG1*(1.-.5*   R(I)**2)/R(I)**3
      H3=R(I)**5-(40.+29.*PR     )*R(I)**3+X6*2.*R(I)*LOGF(R(I))+X1*(4.-2
     10.*R(I)**2)/R(I)**3
      H3=H3/60.
      G4=-X1+(G1-R(I)*H1)/ARG1
      G5=2.*X1*ARG1+R(I)**2*(G2-R(I)*H2)
      G5=G5/(ARG1*R(I)**2)
      G6=(G3-R(I)*H3)+2.*X1*(1.-4.*R(I)**2)*ARG1**4/(15.*R(I)**2)
      G6=G6/ARG1
      G7=(G4-ARG1*G1-.5*X1-.5*PR     *R(I)**2)/R(I)
      G8=(G5-ARG1*G2-PR     -X1/R(I)**2)/R(I)
      G9=-.5*R(I)*(2.+R(I)**2)/ARG1
      G10=-R(I)/ARG1
      G11=ARG1*(15.*R(I)**4+2.*ARG1**4+2.*PR     *ARG1**6)/(30.*R(I)**2)
      G11=(G6-ARG1*G3-G11)/R(I)
      P(I)=2.*CDA*(C1*G7 +C2*G8 +C3*G9 +C4*G10+G11)
      Q(I)=2.*CDA*(C1*G4+C2*G5+G6+(C4-.5*C3*R(I)**2)/ARG1)
      WFLEXA(I)=2.*CDA*(C1*G7+C2*G8+G11)
      UFLEXA(I)=2.*CDA*(C1*G4+C2*G5+G6)
    4 CONTINUE
      UB1=ABSF(Q(1))
      UB2=ABSF(Q(M))
      IF(UB1-UB)47,47,53
   47 Q(1)=0.
   53 IF(UB2-UB)58,58,35
   58 Q(M)=0.
   35 DO 11 I=1,M
      ARG1=1.+GAMMA(I)**2
      ARG1=SQRTF(ARG1)
      ARG2=1./GAMMA(I)
      ARG2=ARG2**2
      EN1(I)=  CS  * ARG1*ARG2*(ARG1**3+A1)/3.
      EN2(I)=CS     *(3.-ARG2*(ARG1**3+A1)/ARG1)/3.
      ARG1=1./GAMMA(I)
      ARG1=ARG1**2
      ARG2=1.+GAMMA(I)**2
      ARG2=SQRTF(ARG2)
      ARG21=1./ARG2
      ARG3=LOGF(GAMMA(I))
      ARG4=ARG1/GAMMA(I)
      ARG5=ARG4*ARG1
      U1(I)=CD     *((X0*GAMMA(I)*LOGF(1.+ARG2)+(X1-X0)*GAMMA(I)*LOGF(GAM
     1MA(I))+A3*GAMMA(I)+X2*GAMMA(I)**3+X3*GAMMA(I)**5-X1/GAMMA(I))*ARG2
     21+A1*GAMMA(I)+X0/GAMMA(I))/3.
      UFLEXS(I)=U1(I)-CD*A3*GAMMA(I)*ARG21/3.
      W1(I)=CD     *((X0*LOGF(1.+ARG2)+(X1-X0)*LOGF(GAMMA(I))+X2*GAMMA(I)
     1**2+X3*GAMMA(I)**4+A3-X1/GAMMA(I)**2)*ARG21+(X4+X5*GAMMA(I)**2+X1/
     2GAMMA(I)**2)*ARG2-X0)/3.
      WFLEXS(I)=W1(I)-CD*A3*ARG21/3.
   11 CONTINUE
      UB1=ABSF(U1(1))
      UB2=ABSF(U1(M))
      IF(UB1-UB)59,59,62
   59 U1(1)=0.
   62 IF(UB2-UB)63,63,80
```

TABLE I (Continued)

```
   63 U1(M)=0.
C
C - - (7) OUTPUT II
C
   80 IF(N)225,225,226
  225 IF(PSI)38,226,227
  226 WRITE OUTPUT TAPE 3,229
      WRITE OUTPUT TAPE 3,231,A1,A3
      WRITE OUTPUT TAPE 3,114
      IF(N)228,228,227
  227 WRITE OUTPUT TAPE 3,230
      WRITE OUTPUT TAPE 3,231,C1,C2,C3,C4
  228 WRITE OUTPUT TAPE 3,107
      IF(PSI)38,33,34
   33 IF(NORM)36,36,61
   36 DO 48 I=1,M
      EN1(I)=EN1(I)/CS
      EN2(I)=EN2(I)/CS
      UFLEXS(I)=UFLEXS(I)/CD
      WFLEXS(I)=WFLEXS(I)/CD
      W1(I)=W1(I)/CD
   48 U1(I)=U1(I)/CD
   61 WRITE OUTPUT TAPE 3,109
      IF(NORM)13,13,14
   13 WRITE OUTPUT TAPE 3,310
      GO TO 15
   14 WRITE OUTPUT TAPE 3,110
   15 WRITE OUTPUT TAPE 3,811,(GAMMR(I),U1(I),W1(I),EN1(I),EN2(I),I=1,M)
      WRITE OUTPUT TAPE 3,107
      IF(NORM)84,84,85
   84 WRITE OUTPUT TAPE 3,348
      GO TO 86
   85 WRITE OUTPUT TAPE 3,148
   86 WRITE OUTPUT TAPE 3,144,(GAMMR(I),UFLEXS(I),WFLEXS(I),I=1,M)
      WRITE OUTPUT TAPE 3,107
      GO TO 10
   34 IF(N)57,57,56
   57 DO 60 I=1,M
      A(I)=A(I)+EN1(I)*CPHI
      B(I)=B(I)+EN2(I)*CPHI
      WFLEXA(I)=WFLEXA(I)+WFLEXS(I)*CPHI
      UFLEXA(I)=UFLEXA(I)+UFLEXS(I)*CPHI
      P(I)=P(I)+W1(I)*CPHI
   60 Q(I)=Q(I)+U1(I)*CPHI
      CNORM=1.
      IF(NORM)44,44,45
   44 DO 46 I=1,M
      A(I)=A(I)/CS
      B(I)=B(I)/CS
      C(I)=C(I)/CS
      G(I)=G(I)/CD
      P(I)=P(I)/CD
      Q(I)=Q(I)/CD
      UFLEXA(I)=UFLEXA(I)/CD
      VFLEXA(I)=VFLEXA(I)/CD
      WFLEXA(I)=WFLEXA(I)/CD
   46 CONTINUE
      CNORM=CD
```

```
 45 IF(NORM)16,16,17
 16 WRITE OUTPUT TAPE 3,303
    GO TO 18
 17 WRITE OUTPUT TAPE 3,103
 18 WRITE OUTPUT TAPE 3,106,(S(I),A(I),B(I),C(I),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(KD-1)42,42,43
 42 IF(NORM)19,19,20
 19 WRITE OUTPUT TAPE 3,304
    GO TO 21
 20 WRITE OUTPUT TAPE 3,104
 21 WRITE OUTPUT TAPE 3,812,(S(I),Q(I),G(I),P(I),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)87,87,88
 87 WRITE OUTPUT TAPE 3,349
    GO TO 89
 88 WRITE OUTPUT TAPE 3,149
 89 WRITE OUTPUT TAPE 3,106,(S(I),UFLEXA(I),VFLEXA(I),WFLEXA(I),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(KD-1)10,43,43
 43 DO 49 I=1,M
    P(I)=P(I)-W1(I) /CNORM
    Q(I)=Q(I)-U1(I)  /CNORM
    WFLEXA(I)=WFLEXA(I)-WFLEXS(I)/CNORM
    UFLEXA(I)=UFLEXA(I)-UFLEXS(I)/CNORM
 49 CONTINUE
    WRITE OUTPUT TAPE 3,142
    IF(NORM)22,22,23
 22 WRITE OUTPUT TAPE 3,439
    GO TO 24
 23 WRITE OUTPUT TAPE 3,239
 24 WRITE OUTPUT TAPE 3,812,(S(I),Q(I),G(I),P(I),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)200,200,201
200 WRITE OUTPUT TAPE 3,440
    GO TO 202
201 WRITE OUTPUT TAPE 3,240
202 WRITE OUTPUT TAPE 3,106,(S(I),UFLEXA(I),VFLEXA(I),WFLEXA(I),I=1,M)
    GO TO 10
 56 DO 3 J=1,N
    DO 3 I=1,M
    V(I,J)=G(I)     *COSF(THETA(J))
    W(I,J)=    SINF(THETA(J))*P(I)              +W1(I)*CPHI
    U(I,J)=    SINF(THETA(J))*Q(I)             +U1(I)*CPHI
    VFLEX(I,J)=VFLEXA(I)*COSF(THETA(J))
    WFLEX(I,J)=WFLEXA(I)*SINF(THETA(J))+WFLEXS(I)*CPHI
    UFLEX(I,J)=UFLEXA(I)*SINF(THETA(J))+UFLEXS(I)*CPHI
    ENR(I,J)=A(I)       *SINF(THETA(J))                +EN1(I)*CPHI
    ENT(I,J)=B(I)       *SINF(THETA(J))                +EN2(I)*CPHI
    ENRT(I,J)=C(I)*COSF(THETA(J))
  3 CONTINUE
    DO 12 J=1,N
 12 THETA(J)=THETA(J)*RAD
    SKIP=0
    CNORM=1.
    IF(NORM)64,64,65
 64 DO 66 I=1,M
    DO 66 J=1,N
```

TABLE I (Continued)

```
      W(I,J)=W(I,J)/CD
      U(I,J)=U(I,J)/CD
      V(I,J)=V(I,J)/CD
      WFLEX(I,J)=WFLEX(I,J)/CD
      UFLEX(I,J)=UFLEX(I,J)/CD
      VFLEX(I,J)=VFLEX(I,J)/CD
      ENR(I,J)=ENR(I,J)/CS
      ENT(I,J)=ENT(I,J)/CS
      ENRT(I,J)=ENRT(I,J)/CS
   66 CONTINUE
      CNORM=CD
      WRITE OUTPUT TAPE 3,330
      GO TO 25
   65 WRITE OUTPUT TAPE 3,130
   25 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
      WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(ENR(I,J),J=1,N),I=1,M)
      WRITE OUTPUT TAPE 3,107
      IF(NORM)26,26,27
   26 WRITE OUTPUT TAPE 3,331
      GO TO 28
   27 WRITE OUTPUT TAPE 3,131
   28 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
      WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(ENT(I,J),J=1,N),I=1,M)
      WRITE OUTPUT TAPE 3,107
      IF(NORM)29,29,37
   29 WRITE OUTPUT TAPE 3,332
      GO TO 41
   37 WRITE OUTPUT TAPE 3,132
   41 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
      WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(ENRT(I,J),J=1,N),I=1,M)
      WRITE OUTPUT TAPE 3,107
      IF(KD-1)82,82,83
   82 IF(NORM)501,501,504
  501 IF(SKIP)502,502,503
  502 WRITE OUTPUT TAPE 3,333
      GO TO 507
  503 WRITE OUTPUT TAPE 3,433
      GO TO 507
  504 IF(SKIP)505,505,506
  505 WRITE OUTPUT TAPE 3,133
      GO TO 507
  506 WRITE OUTPUT TAPE 3,233
  507 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
      WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(U(I,J),J=1,N),I=1,M)
      WRITE OUTPUT TAPE 3,107
      IF(NORM)508,508,511
  508 IF(SKIP)509,509,510
  509 WRITE OUTPUT TAPE 3,334
      GO TO 514
  510 WRITE OUTPUT TAPE 3,434
      GO TO 514
  511 IF(SKIP)512,512,513
  512 WRITE OUTPUT TAPE 3,134
      GO TO 514
```

TABLE I (Continued)

```
513 WRITE OUTPUT TAPE 3,234
514 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
    WRITE OUTPUT TAPE 3,114
    WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(V(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)515,515,518
515 IF(SKIP)516,516,517
516 WRITE OUTPUT TAPE 3,335
    GO TO 521
517 WRITE OUTPUT TAPE 3,435
    GO TO 521
518 IF(SKIP)519,519,520
519 WRITE OUTPUT TAPE 3,135
    GO TO 521
520 WRITE OUTPUT TAPE 3,235
521 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
    WRITE OUTPUT TAPE 3,114
    WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(W(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)522,522,525
522 IF(SKIP)523,523,524
523 WRITE OUTPUT TAPE 3,350
    GO TO 528
524 WRITE OUTPUT TAPE 3,436
    GO TO 528
525 IF(SKIP)526,526,527
526 WRITE OUTPUT TAPE 3,150
    GO TO 528
527 WRITE OUTPUT TAPE 3,236
528 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
    WRITE OUTPUT TAPE 3,114
    WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(UFLEX(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)529,529,532
529 IF(SKIP)530,530,531
530 WRITE OUTPUT TAPE 3,351
    GO TO 535
531 WRITE OUTPUT TAPE 3,437
    GO TO 535
532 IF(SKIP)533,533,534
533 WRITE OUTPUT TAPE 3,151
    GO TO 535
534 WRITE OUTPUT TAPE 3,237
535 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
    WRITE OUTPUT TAPE 3,114
    WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(VFLEX(I,J),J=1,N),I=1,M)
    WRITE OUTPUT TAPE 3,107
    IF(NORM)536,536,539
536 IF(SKIP)537,537,538
537 WRITE OUTPUT TAPE 3,352
    GO TO 542
538 WRITE OUTPUT TAPE 3,438
    GO TO 542
539 IF(SKIP)540,540,541
540 WRITE OUTPUT TAPE 3,152
    GO TO 542
541 WRITE OUTPUT TAPE 3,238
542 WRITE OUTPUT TAPE 3,FMT1,(THETA(J),J=1,N)
```

TABLE I (Continued)

```
      WRITE OUTPUT TAPE 3,114
      WRITE OUTPUT TAPE 3,FMT2,(GAMMR(I),(WFLEX(I,J),J=1,N),I=1,M)
      WRITE OUTPUT TAPE 3,107
      IF(SKIP)215,215,10
  215 IF(KD-1)10,83,83
   83 DO 224 J=1,N
      DO 223 I=1,M
      UFLEX(I,J)=UFLEX(I,J)-U1(I)/CNORM
      WFLEX(I,J)=WFLEX(I,J)-W1(I)/CNORM
      U(I,J)=U(I,J)-U1(I)/CNORM
  223 W(I,J)=W(I,J)-W1(I)/CNORM
  224 CONTINUE
      WRITE OUTPUT TAPE 3,142
      SKIP=1
      GO TO 82
C
C - - (8) RETURN TO  (1)
C
  101 FORMAT(8I5)
  102 FORMAT(5E14.8)
  103 FORMAT(116H                          R(IN.)                      NR(LB./IN.)
     1                NTHETA(LB./IN.)           NRTHETA(LB./IN.)         ////)
  104 FORMAT (104H                         R(IN.)                      U(IN.)
     1                 V(IN.)                    W(IN.)       ////)
  105 FORMAT (4F15.9)
  106 FORMAT(F25.4,3F25.8)
  107 FORMAT (1H1)
  108 FORMAT(4F24.5,F18.2)
  109 FORMAT (75H  THE LOADING AS WELL AS THE DEFORMATION OF THE SHELL I
     1S AXISYMMETRIC       ////)
  110 FORMAT(122H                      R(IN.)                      U(IN.)
     1             W(IN.)                   NR(LB./IN.)           NTHETA(LB./IN.
     2)////)
  111 FORMAT(5F24.5)
  112 FORMAT(120H       WEIGHT DENSITY(LB./IN.2)     YOUNGS MODULUS(LB./IN.
     12)       THICKNESS(IN.)        FOCAL LENGTH(IN.)      POISSONS RATIO/
     2///)
  113 FORMAT(116H                        R1(IN.)                     R2(IN.)
     1           R3(IN.)                  R4(IN.)                 PSI(DEG.)////)
  114 FORMAT (////)
  115 FORMAT ( 50H  THE SHELL IS FIXED TANGENTIALLY AT BOTH EDGES   ///)
  116 FORMAT ( 70H   THE SHELL IS FIXED TANGENTIALLY AT R2 AND IS FREE A
     1T R1          ////)
  117 FORMAT(51H                    THETA3(DEG.)             THETA4(DEG.)////)
  118 FORMAT(75H    THE SHELL IS CLOSED AT THE APEX AND IS FIXED TANGENTI
     1ALLY AT R2           ////)
  119 FORMAT(2F24.2)
  130 FORMAT(50H             -----STRESS RESULTANT NR(LB./IN.)----- /////)
  131 FORMAT(53H             -----STRESS RESULTANT NTHETA(LB./IN.)-----///
     1//)
  132 FORMAT(54H             -----STRESS RESULTANT NRTHETA(LB./IN.)-----//
     1///)
  133 FORMAT(50H             -----DISPLACEMENT U(IN.)----            /////)
  134 FORMAT(50H             -----DISPLACEMENT V(IN.)-----           /////)
  135 FORMAT(50H             -----DISPLACEMENT W(IN.)-----           /////)
  139 FORMAT(51H ---NORMALIZED RESULTS BY MEMBRANE ANALYSIS          ////)
  140 FORMAT(51H ---UN-NORMALIZED RESULTS BY MEMBRANE ANALYSIS       ////)
  142 FORMAT(95H THE DISTORTION OF THE SHELL GIVEN BELOW IS MEASURED REL
```

TABLE I (Continued)

```
   1ATIVE TO THAT OF THE FACE-UP POSITION    ////)
143 FORMAT (55H  THE PROGRAM DOES NOT ACCEPT A NEGATIVE POINTING ANGLE
   1////)
144 FORMAT(F24.4,2F24.8)
148 FORMAT(116H                        R(IN.)             UFLEX(IN.)
   1        WFLEX(IN.)                                                ////)
149 FORMAT(116H                        R(IN.)             UFLEX(IN.)
   1        VFLEX(IN.)                   WFLEX(IN.)                   ////)
150 FORMAT(50H           ----DISPLACEMENT UFLEX(IN.)----          /////)
151 FORMAT(50H           ----DISPLACEMENT VFLEX(IN.)----          /////)
152 FORMAT(50H           ----DISPLACEMENT WFLEX(IN.)----          /////)
160 FORMAT(56H  A HOMOGENEOUS PARABOLOIDAL SHELL SUBJECTED TO GRAVITY
   1////)
161 FORMAT(56H  A LAMINAR PARABOLOIDAL SHELL SUBJECTED TO GRAVITY
   1////)
162 FORMAT(56H  A SANDWICH PARABOLOIDAL SHELL SUBJECTED TO GRAVITY
   1////)
163 FORMAT(116H  STRETCHING STIFFNESS(LB/IN**2)  WEIGHT DENSITY(LB/IN*
   1*2)  FOCAL LENGTH(IN.)       POISSONS RATIO                       ////)
164 FORMAT(116H  YOUNGS MODULUS(LB/IN**2)        CORE THICKNESS(IN.)
   1      SKIN THICKNESS(IN.)    RHO OF SKIN(LB/IN**3)                ////)
165 FORMAT(116H    RHO OF CORE(LB/IN**3)          WEIGHT DENSITY(LB/IN*
 · 1*2)  FOCAL LENGTH(IN.)       POISSONS RATIO                       ////)
205 FORMAT(12A6)
229 FORMAT(19X,2HA1,23X,2HA2,////)
230 FORMAT(19X,2HC1,23X,2HC2,23X,2HC3,23X,2HC4,////)
231 FORMAT(4F25.6)
233 FORMAT(50H           -----DISPLACEMENT U TILDE(IN.)----       /////)
234 FORMAT(50H           -----DISPLACEMENT V TILDE(IN.)----       /////)
235 FORMAT(50H           -----DISPLACEMENT W TILDE(IN.)----       /////)
236 FORMAT(50H           -----DISPLACEMENT UFLEX TILDE(IN.)---- /////)
237 FORMAT(50H           -----DISPLACEMENT VFLEX TILDE(IN.)---- /////)
238 FORMAT(50H           -----DISPLACEMENT WFLEX TILDE(IN.)---- /////)
239 FORMAT(116H                        R(IN.)             U TILDE(IN.)
   1           V TILDE(IN.)              W TILDE(IN.)                 ////)
240 FORMAT(116H             .          R(IN.)          UFLEX TILDE(IN.)
   1     UFLEX TILDE(IN.)         WFLEX TILDE(IN.)                    ////)
303 FORMAT(116H                        GAMMA                      NR*
   1             NTHETA*                 NRTHETA*                     ////)
304 FORMAT (104H                       GAMMA                    U*
   1             V*                        W*         ////)
310 FORMAT(121H                        GAMMA                    U*
   1            W*                   · NR*                      NTHETA*
   2////)
330  FORMAT(75H           ----NORMALIZED STRESS RESULTANT NR*(DIMENSION
   1LESS)----            /////)
331 FORMAT(75H           ----NORMALIZED STRESS RESULTANT NTHETA*(DIMEN
   1SIONLESS)----        /////)
332 FORMAT(75H           ----NORMALIZED STRESS RESULTANT NRTHETA*(DIME
   1NSIONLESS)----       /////)
333 FORMAT(60H           ----NORMALIZED DISPLACEMENT U*(DIMENSIONLESS)
   1----/////)
334 FORMAT(60H           ----NORMALIZED DISPLACEMENT V*(DIMENSIONLESS)
   1----/////)
335 FORMAT(60H           ----NORMALIZED DISPLACEMENT W*(DIMENSIONLESS)
   1----/////)
348 FORMAT(116H                        GAMMA                  UFLEX*
   1          WFLEX*                                                  ////)
```

TABLE I (Continued)

```
 349 FORMAT(116H                                GAMMA                   UFLEX*
    1             VFLEX*                             WFLEX*                     ////)
 350   FORMAT(65H               ----NORMALIZED DISPLACEMENT UFLEX*(DIMENSIONL
    1ESS----    //////)
 351   FORMAT(65H               ----NORMALIZED DISPLACEMENT VFLEX*(DIMENSIONL
    1ESS----    //////)
 352   FORMAT(65H               ----NORMALIZED DISPLACEMENT WFLEX*(DIMENSIONL
    1ESS----    //////)
 433 FORMAT(75H                 ----NORMALIZED DISPLACEMENT U TILDE*(DIMENSIO
    1NLESS)----             /////)
 434 FORMAT(75H                 ----NORMALIZED DISPLACEMENT V TILDE*(DIMENSIO
    1NLESS)----             /////)
 435 FORMAT(75H                 ----NORMALIZED DISPLACEMENT W TILDE*(DIMENSIO
    1NLESS)----             /////)
 436 FORMAT(75H                 ----NORMALIZED DISPLACEMENT UFLEX TILDE* (DIM
    1ENSIONLESS)----        /////)
 437 FORMAT(75H                 ----NORMALIZED DISPLACEMENT VFLEX TILDE* (DIM
    1ENSIONLESS)----        /////)
 438 FORMAT(75H                 ----NORMALIZED DISPLACEMENT WFLEX TILDE* (DIM
    1ENSIONLESS)----        /////)
 439 FORMAT(116H                                GAMMA                   U TILDE*
    1                 V TILDE*                        W TILDE*                ////)
 440 FORMAT(116H                                GAMMA               UFLEX TILDE*
    1          VFLEX TILDE*                 WFLEX TILDE*                      ////)
 811 FORMAT(F24.4,E24.6,3F24.8)
 812 FORMAT(F25.4,E25.6,2F25.8)
     END
*    DATA
```

1. Input

The input to our program as indicated by step (1) in Fig. 12 consists of a number of records prestored on tape A2.

Record 1 Control Parameters (6I5)

This line contains six non-negative fixed-point variables (integers).

| M | N | KE | NORM | KD | KP |
|---|---|---|---|---|---|

M ($\leqslant 150$) Number of (equally spaced) $\gamma$ – tabular points between R3 and R4 (R3 and R4 are to be defined later) for any fixed value of $\theta$.

N ($\leqslant 10$) Number of (equally spaced) $\theta$ – tabular points between THETA3 and THETA4 (THETA3 and THETA4 are to be defined later) for any fixed value of $\gamma$. If only those quantities with superscript zero are desired [see Eqs. (II-85) and (II-86)], set N equal to zero.

KE Control parameter specifying the edge conditions at R1 and R2 (R1 and R2 will be defined later. They are not to be confused with $r_1$ and $r_2$ which are the lower and upper edge of the shell).

KE < 1 Shell is fixed tangentially at both edges.

KE = 1 Shell is fixed tangentially at R2 and is free at R1.

KE > 1 Shell is closed at the apex and is supported tangentially at R2.

NORM Control parameter specifying whether normalized results are desired.

NORM = 0 Normalized results.

NORM > 0 Un-normalized results.

KD Control parameter specifying the type of distortion to be given in the output.

KD < 1 u, v, w, and the corresponding $u_{flex}$, $v_{flex}$, $w_{flex}$ are given in the output.

KD > 1 $\tilde{u}$, $\tilde{v}$, $\tilde{w}$, and the corresponding $\tilde{u}_{flex}$, $\tilde{v}_{flex}$, $\tilde{w}_{flex}$ are given in the output.

KD = 1 u, v, w, $\tilde{u}$, $\tilde{v}$, $\tilde{w}$, as well as the corresponding six flex quantities.

If N = 0, a superscript zero should be added to these displacement quantities.

KP Control parameter specifying the type of shell under consideration. Set

KP < 1 if the shell is homogeneous,

KP > 1 if the shell is a sandwiched construction with a very soft core,

KP = 1 if the shell is laminar in that its material properties vary across the thickness in a manner other than previously mentioned.

Record 2 Geometrical Parameters (4F15.9)

This line contains four non-negative F-type floating point variables.

| R1 | R2 | R3 | R4 |
|---|---|---|---|

R1 (in.) Value of r at the free edge of the shell. If the shell is closed at the apex, then R1 = 0. If the shell is fixed tangentially at both edges, then set R1 equal to the value of r at the lower edge.

R2 (in.) Value of r at the tangentially fixed edge. If both edges of the shell are fixed tangentially, set R2 equal to the value of r at the upper edge.

R3 (in.) Smallest value of r at which the stresses and distortion of the shell are to be given in the output. Generally, this is the same as $r_1$. However, if the shell is closed at the apex, it is usually wise to avoid calculating the limiting value of the desired output at the apex by setting R3 > 0.

R4 (in.) Largest value of r at which the stresses and distortion of the shell are to be given in the output. Generally, this is the same as $r_2$.

Record 3 Geometrical and Material Parameters (3F15.9)

This line contains three non-negative F-type floating point variables.

| F | PR | PSI |
|---|---|---|

F Focal length of the paraboloidal surface (in.).

PR Poisson's ratio.

PSI Pointing angle $\psi$ in fractions of $\pi$ (e.g., if $\psi = 45°$, then psi = 0.25).

Record 4 Geometrical and Material Parameters (5E14.8)

KP < 1 This line contains three E-type floating point variables.

| RHO | H | E |
|---|---|---|

RHO Volume weight density of shell (lb/in.$^3$).

H Shell thickness (in.).

E Young's modulus (lb/in.$^2$).

KP > 1 This line contains five E-type floating point variables.

| RHS | RHC | T | H | E |
|---|---|---|---|---|

RHS Volume weight density of skin (lb/in.$^3$).

RHC Volume weight density of core (lb/in.$^3$).

T Thickness of skin (in.).

H Thickness of core (in.).

E Young's modulus of skin (lb/in.$^2$).

KP = 1  This line contains two E-type floating point variables.

| RHO | CC |
|---|---|

RHO  Surface weight density of shell (lb/in.$^2$).

CC  Extensional stiffness of shell.

Record 5  Geometrical Parameters (2F15.9)

This line contains two F-type floating point variables.

| THETA3 | THETA4 |
|---|---|

THETA3  Smallest value of $\Theta$ in fractions of $\pi$ at which the stresses and distortion of the shell are to be given in the output.

THETA4  Largest value of $\Theta$ in fractions of $\pi$ at which the stresses and distortion of the shell are to be given in the output.

Record 6  Variable Format Statement I

This line provides a format statement for listing the set of $\Theta$-tabular points (see also Sec. III-B-2), for example,

| (19H R(IN.)/THETA(DEG.), NF11.2) |
|---|

.

Record 7  Variable Format Statement II

This line provides a format statement for the values of each physical quantity at different positions (see also Sec. III-B-2), for example,

| (F19.4, NF11.6) |
|---|

.

Remark:– If N = 0 or if PSI = 0, then the input must consist of only the first four records in the preceding list. Additional records will be treated as a new set of input upon return. For the purpose of an Express Run, each of these records is punched out on one card. A complete set of input will then consist of seven cards (or four cards if N = 0 or if $\psi$ = 0). It is to be placed after the DATA card which in turn follows the object deck. Additional runs can be made by merely placing additional sets of input after the first set.

If some restriction on the input is violated, the program returns to step (1) to take in a new set of data. It may or may not give a statement pointing out the source of trouble.

2. Output

The first part of the output, given by step (2) in Fig. 12, reproduces for the record the input to the program. It states the problem, the edge conditions, the various geometrical and material parameters and whether the numerical results have been normalized.

The second part of the output, given by step (7) in Fig. 12, gives the values of the six physical quantities, $N_r$, $N_\Theta$, $N_{r\Theta}$, u, v, and w at the tabular points. If $\psi = 0$, the behavior of the shell is axisymmetric. Thus, $v \equiv N_{r\Theta} \equiv 0$, and $N_r$, $N_\Theta$, u, and w are functions of $\gamma$ only. The output in this case is given in one block of five columns of numbers. The first column lists the set of $\gamma$-tabular points. The next four columns list the values of u, w, $N_r$, and $N_\Theta$ in that order, corresponding to these values of $\gamma$.

If $\psi \neq 0$, the stress resultants and the displacements generally depend on both $\gamma$ and $\Theta$. The output for each of these quantities is arranged in a rectangular block so that each column of data

corresponds to the values of the quantity for fixed value of $\Theta$ and for different values of $\gamma$, and each row corresponds to the values of the quantity for a fixed $\gamma$ and for different values of $\Theta$. Inasmuch as N is not constant, the number of columns may vary for different runs. Therefore, if $N > 0$, two variable format statements are needed for the output. The first of these is used to list all possible values of $\Theta$ as the first record of the block. The second is used repeatedly to list records of (N + 1) numbers. The first number of each record is a value of $\gamma$ while the remaining numbers are the N values of the physical quantity in question corresponding to the N positions given by this value of r and the N $\Theta$-tabular points listed in the first record. Examples given in a later section will clarify this discussion. Having listed all the stress and displacement quantities, the next block of data gives the maximum values (in modulus) of $u_{flex}$, $v_{flex}$, and $w_{flex}$ or $\tilde{u}_{flex}$, $\tilde{v}_{flex}$, and $\tilde{w}_{flex}$ for the set of tabular $\gamma$'s.

If N = 0, so that only the zero-superscripted quantities are requested, this second part of the output will be arranged in three blocks of data each consisting of four columns. The first of these columns in each block lists the set of $\gamma$-tabular points. The remaining three columns of the first block list $N_r^o$, $N_\Theta^o$, and $N_{r\Theta}^o$, those of the second block list $u^o$, $v^o$, and $w^o$, and those of the third list $u_{flex}^o$, $v_{flex}^o$, and $w_{flex}^o$ for the given set of tabular points.

The above discussion applies only when KD < 1. For KD > 1, $\tilde{u}^o$, $\tilde{v}^o$, and $\tilde{w}^o$ and the corresponding $\tilde{u}_{flex}^o$, $\tilde{v}_{flex}^o$, and $\tilde{w}_{flex}^o$ will take the place of $u^o$, $v^o$, and $w^o$ and the corresponding $u_{flex}^o$, $v_{flex}^o$, and $w_{flex}^o$. If KD = 1, additional blocks of data will appear for the obvious reason. Keeping in mind the discussion in the earlier sections, the output in this case is self-explanatory.

The numerical results in the output are generally printed in the F-type floating point format. There are a few exceptions due to the size of the numbers involved. In these exceptional cases, the E-type floating point format is used.

### 3. Operational Information

The main program is coded in FORTRAN language for a 32K IBM 7090 (and IBM 7094) and is compiled by the FORTRAN II compiler. It requires no routines other than those appearing as library routines on the Lincoln Laboratory library tape. It should be cautioned that these may be different from the routines under the same name used elsewhere. The correspondence between logical tape units and machine tape units is also different at Lincoln Laboratory. Approximately 11,000 storage locations have not been touched by the program. Neither sense switch nor sense light is used. Some of the subscripted variables and their analytical counterparts are presented in Table II.

It should be understood that if normalized results are requested, the stress and displacement expressions in the third column of Table II should be replaced by the corresponding starred quantities.

## C. EXAMPLES

### 1. Axisymmetric Deformations

Consider a sandwich shell supported at the lower edge $r_1$ in a face-up position ($\psi = 0$) with

$$\begin{aligned} rhc &= 0.01 \text{ lb/in.}^3 \\ rhs &= 0.1 \text{ lb/in.}^3 \\ E &= 10^7 \text{ lb/in.}^2 \\ \nu &= 0.3 \end{aligned}$$

TABLE II

SOME SUBSCRIPTED VARIABLES AND THEIR ANALYTICAL COUNTERPARTS

| Subscripted Variable | Dimensionality | Analytical Expression |
|---|---|---|
| U | 150 × 10 | $u$ |
| V | 150 × 10 | $v$ |
| W | 150 × 10 | $w$ |
| UFLEX | 150 × 10 | $u_{flex}$ |
| VFLEX | 150 × 10 | $v_{flex}$ |
| WFLEX | 150 × 10 | $w_{flex}$ |
| ENR | 150 × 10 | $N_r$ |
| ENT | 150 × 10 | $N_\Theta$ |
| ENRT | 150 × 10 | $N_{r\Theta}$ |
| U1 | 150 | $u^s$ |
| W1 | 150 | $w^s$ |
| G | 150 | $v^a$ |
| P | 150 | $w^a$ |
| Q | 150 | $u^a$ |
| EN1 | 150 | $N_r^s$ |
| EN2 | 150 | $N_\Theta^s$ |
| A | 150 | $N_r^a$ |
| B | 150 | $N_\Theta^a$ |
| C | 150 | $N_{r\Theta}^a$ |
| UFLEXA | 150 | $u_{flex}^a$ |
| VFLEXA | 150 | $v_{flex}^a$ |
| WFLEXA | 150 | $w_{flex}^a$ |
| UFLEXS | 150 | $u_{flex}^s$ |
| VFLEXS | 150 | $v_{flex}^s$ |
| WFLEXS | 150 | $w_{flex}^s$ |
| GAMMR | 150 | $r$ |
| GAMMA | 150 | $\gamma$ |
| THETA | 150 | $\Theta$ |

$r_1 = 100$ in.

$r_2 = 300$ in.

$f = 500$ in.

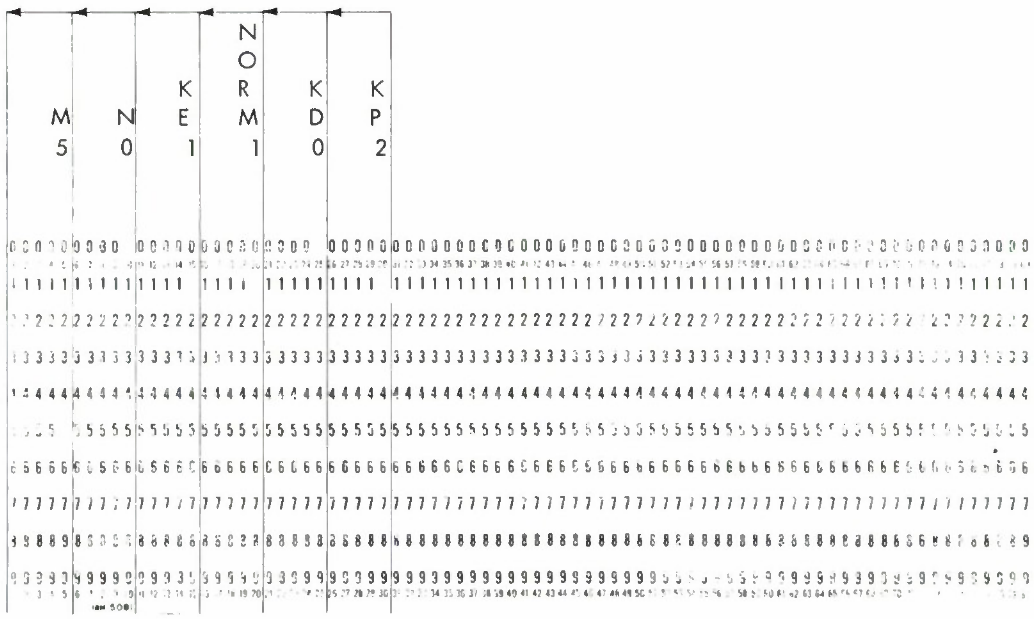
$h = 0.5$ in.

$t = 0.1$ in.

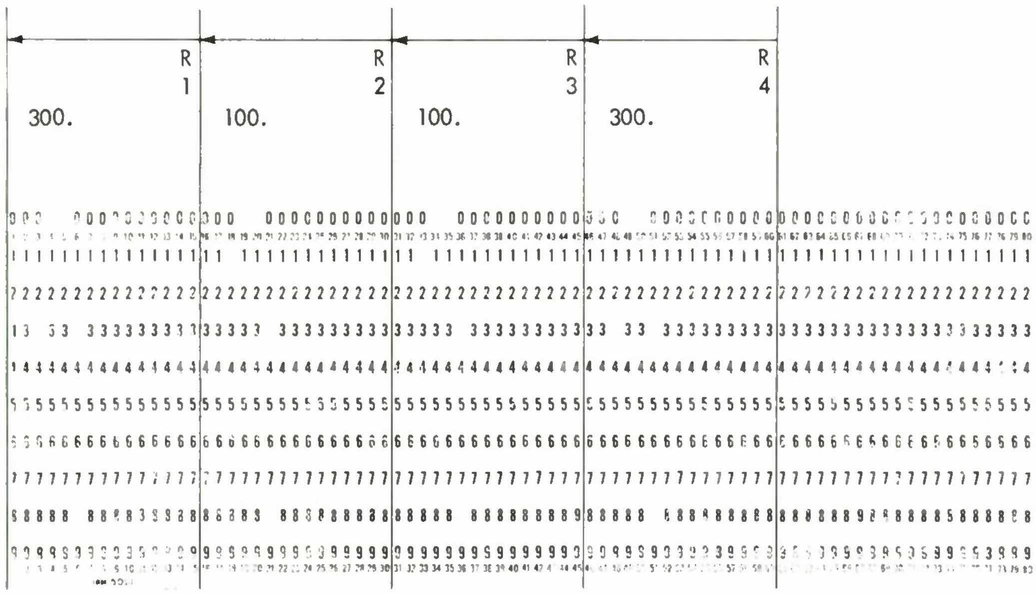
Since the deformation is axisymmetric, we set $N = 0$ and omit records 5, 6, and 7. For the purpose of illustration, we consider only five $\gamma$-tabular points ($M = 5$). The output will not be normalized so that we get the actual physical quantities. Insofar as displacement quantities are concerned, we want only u and w (and of course the corresponding flex displacements).

## INPUT

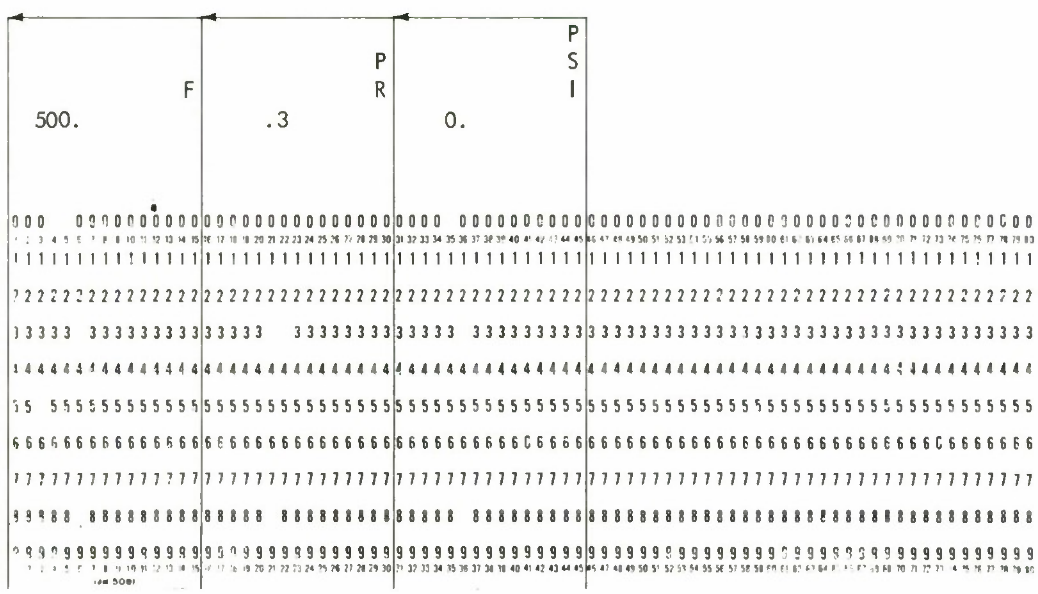
Record 1 Control Parameters (6I5)



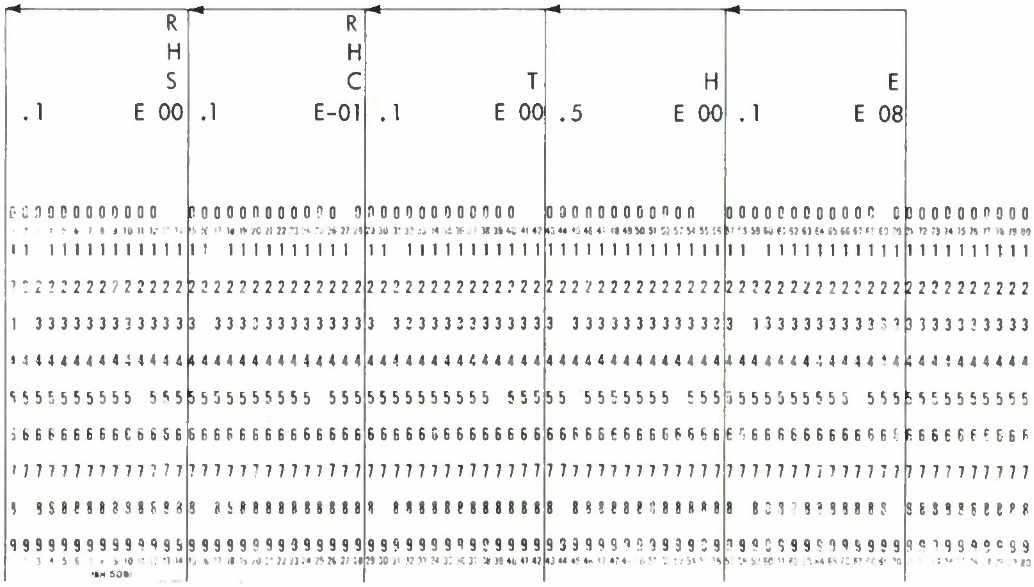
Record 2 Geometrical Parameters (4F15.9)




Record 3 Geometrical and Material Parameters (3F15. 9)



Record 4 Geometrical and Material Parameters (5E14.8)




Remark:– Since PSI is zero, the last three records must be omitted according to input instructions.

The computer output is presented in Table III.

2. Unsymmetric Deformations

Consider a homogeneous shell supported by both edges and oriented in such a way that $\psi \neq 0$. We have

$$E = 10^7 \text{ lb/in.}^2$$
$$\nu = 0.3$$
$$r_1 = 100 \text{ in.}$$
$$r_2 = 300 \text{ in.}$$
$$f = 500 \text{ in.}$$
$$h = 0.01 \text{ in.}$$
$$\text{rho} = 0.1 \text{ lb/in.}^3$$

We now want normalized outputs for seven $\Theta$-tabular points in the interval $-(\pi/2) \leqslant \Theta \leqslant (\pi/2)$ and again five $\gamma$-tabular points in the interval $\gamma_1 \leqslant \gamma \leqslant \gamma_2$. Insofar as displacements are concerned, we will limit ourselves to $\tilde{u}$, $\tilde{v}$, $\tilde{w}$, and the corresponding flex quantities. (In this case, that quantity which is listed in the output as the weight density is rho × h.)

TABLE III

COMPUTER OUTPUT (AXISYMMETRIC DEFORMATIONS)

A SANDWICH PARABOLOIDAL SHELL SUBJECTED TO GRAVITY

---UN-NORMALIZED RESULTS BY MEMBRANE ANALYSIS

THE SHELL IS FIXED TANGENTIALLY AT R2 AND IS FREE AT R1

| YOUNGS MODULUS(LB/IN**2) | CORE THICKNESS(IN.) | SKIN THICKNESS(IN.) | RHO OF SKIN(LB/IN**3) |
|---|---|---|---|
| 10000000.00000 | 0.50000 | 0.10000 | 0.10000 |

| RHO OF CORE(LB/IN**3) | WEIGHT DENSITY(LB/IN**2) | FOCAL LENGTH(IN.) | POISSONS RATIO |
|---|---|---|---|
| 0.01000 | 0.02400 | 500.00000 | 0.30000 |

| R1(IN.) | R2(IN.) | R3(IN.) | R4(IN.) | PSI(DEG.) |
|---|---|---|---|---|
| 300.00000 | 100.00000 | 100.00000 | 300.00000 | 0. |

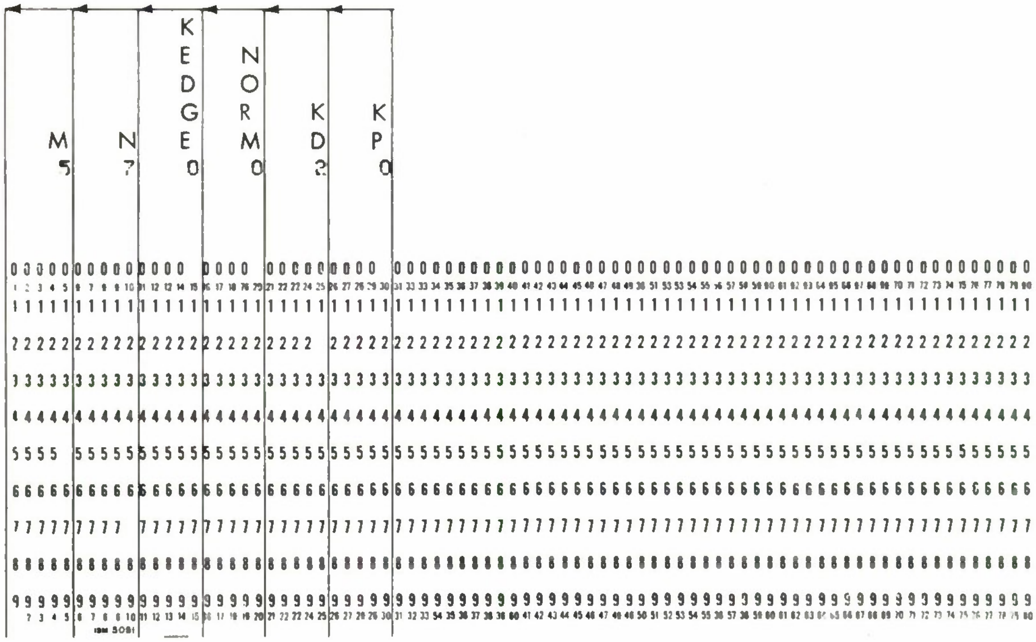
THE LOADING AS WELL AS THE DEFORMATION OF THE SHELL IS AXISYMMETRIC

| R(IN.) | U(IN.) | W(IN.) | NR(LB./IN.) | NTHETA(LB./IN.) |
|---|---|---|---|---|
| 100.0000 | 0. | -0.07614445 | -98.85535336 | 121.87658787 |
| 150.0000 | -0.594946E-02 | -0.07597028 | -37.41095829 | 60.58773375 |
| 200.0000 | -0.106289E-01 | -0.07553648 | -15.78601956 | 39.17886400 |
| 250.0000 | -0.147280E-01 | -0.07489289 | -5.64614344 | 29.31401658 |
| 300.0000 | -0.184626E-01 | -0.07407248 | 0. | 23.99999928 |

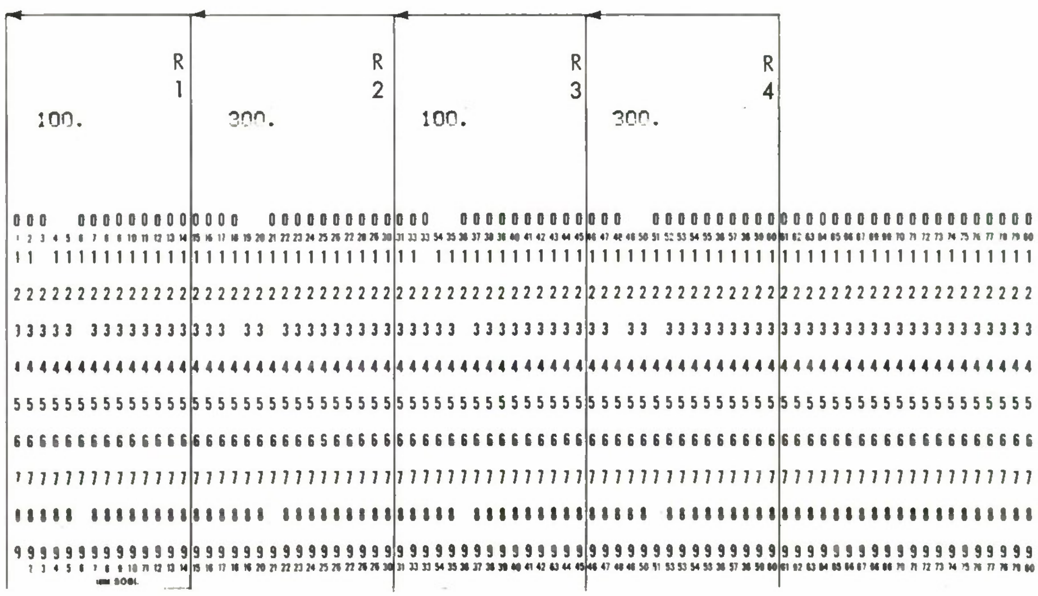
| R(IN.) | UFLEX(IN.) | WFLEX(IN.) |
|---|---|---|
| 100.0000 | 0.00755791 | -0.00056531 |
| 150.0000 | 0.00531790 | -0.00005453 |
| 200.0000 | 0.00426735 | -0.00105540 |
| 250.0000 | 0.00369410 | -0.00120465 |
| 300.0000 | 0.00336318 | -0.00131774 |

<u>INPUT</u>

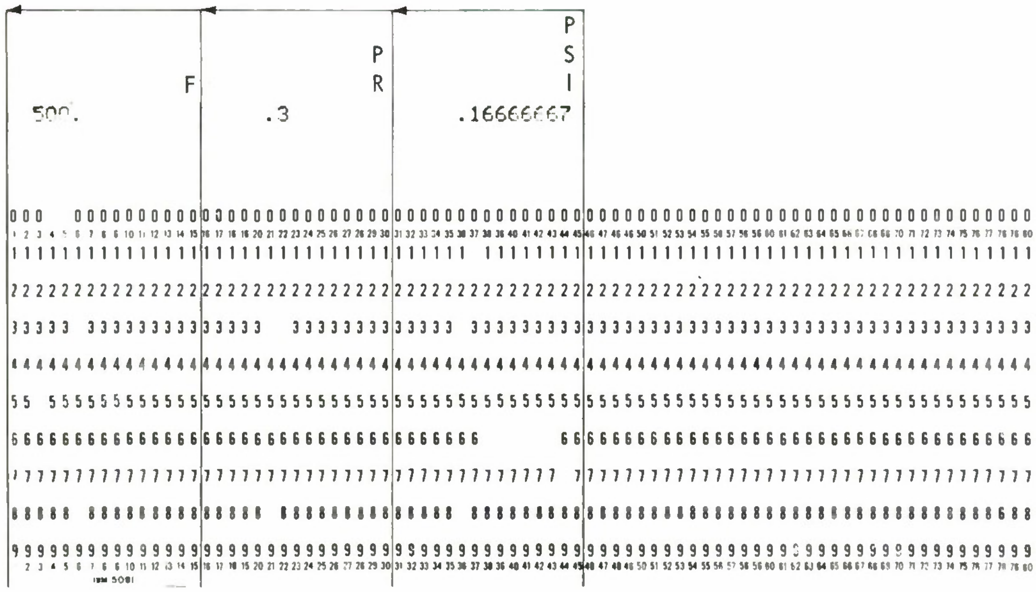
Record 1 Control Parameters (6I5)



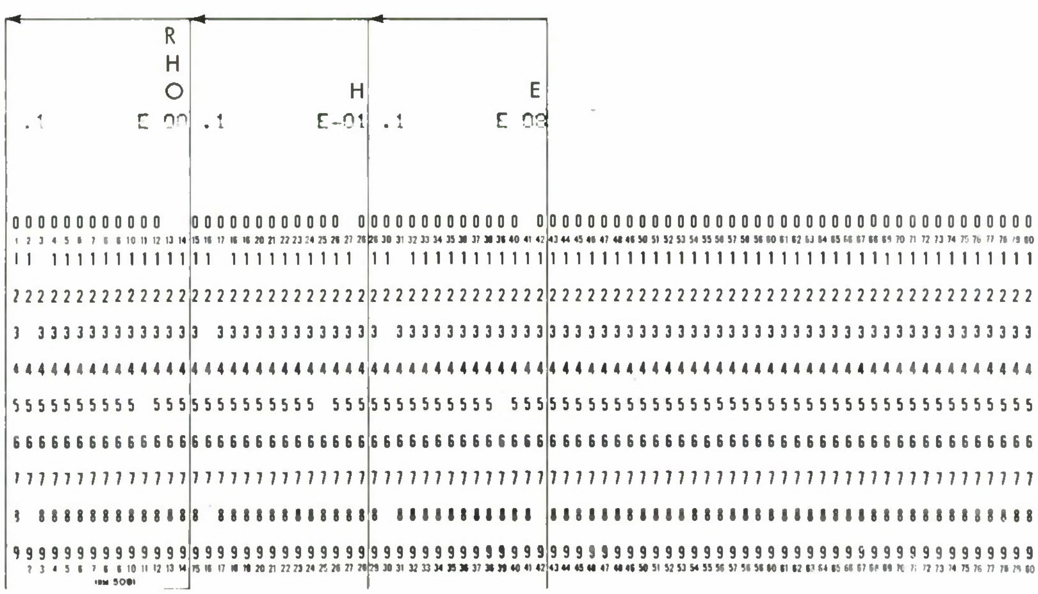

Record 2 Geometrical Parameters (4F15. 9)



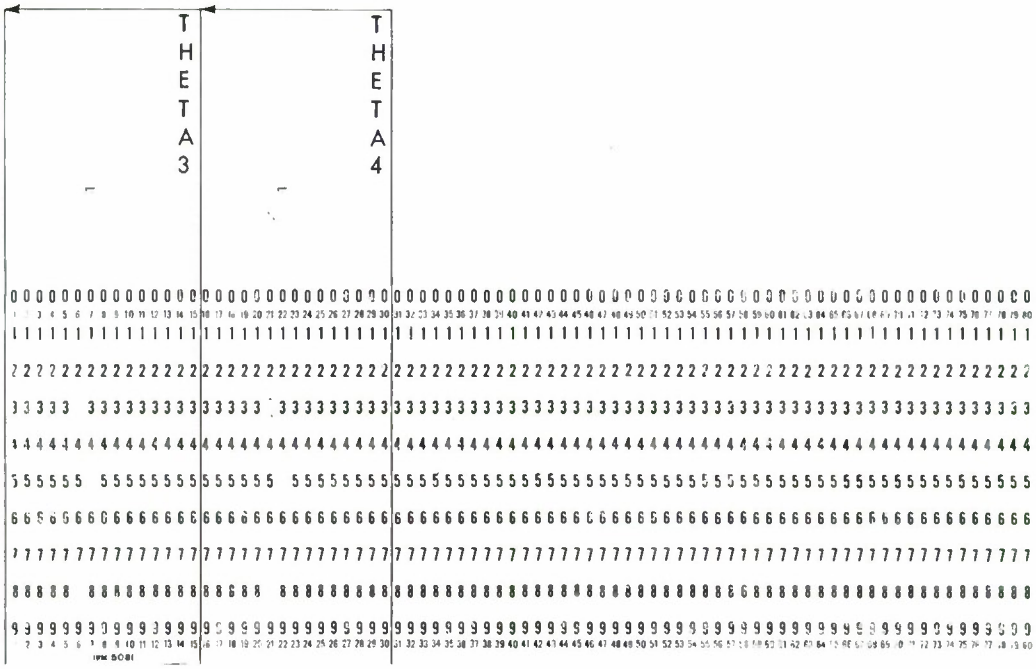
Record 3 Geametrical and Material Parameters (3F15.9)




Recard 4 Geametrical and Material Parameters (3E14.8)

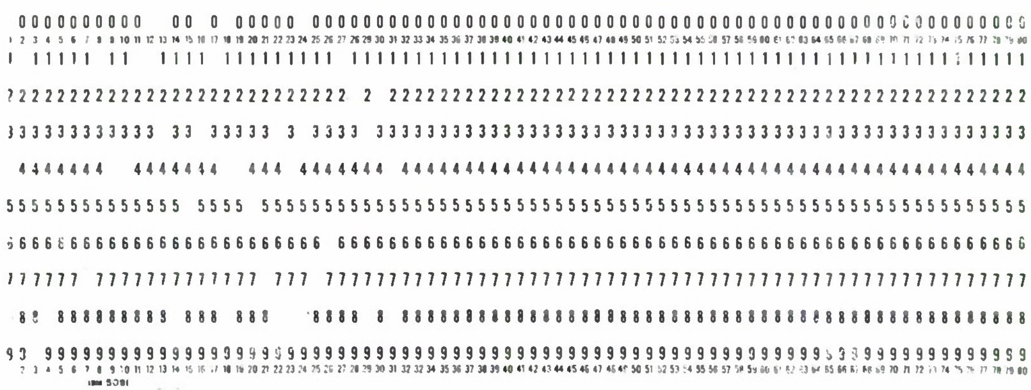


Record 5 Geometrical Parameters (2F15.9)



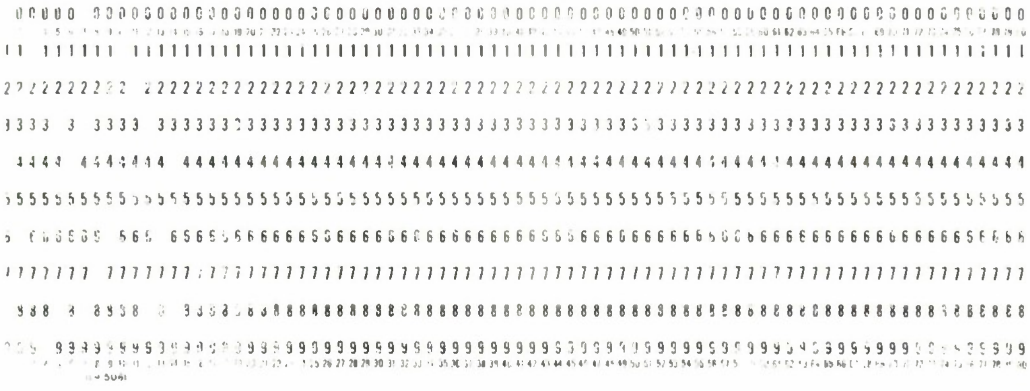

Record 6 Variable Format Statement I (FMT1)

(19H GAMMA/THETA(DEG.), 7F12.2)



Recard 7 Variable Farmat Statement II (FMT2)

(F19.4,7F12.6)



The computer output is presented in Table IV.

TABLE IV

COMPUTER OUTPUT (UNSYMMETRIC DEFORMATIONS)

```
A HOMOGENECUS PARABOLOIDAL SHELL SUBJECTED TO GRAVITY

---NORMALIZED RESULTS BY MEMBRANE ANALYSIS

THE SHELL IS FIXED TANGENTIALLY AT BOTH EDGES

   WEIGHT DENSITY(LB./IN.2)   YOUNGS MODULUS(LB./IN.2)   THICKNESS(IN.)   FOCAL LENGTH(IN.)   POISSONS RATIO

            0.00100          10000000.00000               0.01000          500.00000             0.30000

            R1(IN.)              R2(IN.)                  R3(IN.)           R4(IN.)            PSI(DEG.)

          100.00000            300.00000                100.00000         300.00000            30.00

         THETA3(DEG.)          THETA4(DEG.)

           -90.00                90.00
```

TABLE IV (Continued)

----NORMALIZED STRESS RESULTANT NR•(DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | 0.287550 | 0.304694 | 0.351534 | 0.415518 | 0.479503 | 0.526343 | 0.543487 |
| 0.1500 | 0.355326 | 0.365469 | 0.393181 | 0.431036 | 0.468892 | 0.496604 | 0.506747 |
| 0.2000 | 0.394594 | 0.400773 | 0.417654 | 0.440713 | 0.463773 | 0.480654 | 0.486833 |
| 0.2500 | 0.424292 | 0.427715 | 0.437066 | 0.449839 | 0.462612 | 0.471962 | 0.475385 |
| 0.3000 | 0.453388 | 0.451637 | 0.455048 | 0.459709 | 0.464369 | 0.467780 | 0.469029 |

----NORMALIZED STRESS RESULTANT NTHETA•(DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | 0.531323 | 0.521047 | 0.452972 | 0.454621 | 0.416270 | 0.388195 | 0.377919 |
| 0.1500 | 0.443519 | 0.443647 | 0.443996 | 0.444474 | 0.444951 | 0.445301 | 0.445429 |
| 0.2000 | 0.386608 | 0.394065 | 0.414435 | 0.442262 | 0.470090 | 0.490460 | 0.497917 |
| 0.2500 | 0.341691 | 0.355217 | 0.392178 | 0.442648 | 0.493126 | 0.530079 | 0.543604 |
| 0.3000 | 0.302825 | 0.321776 | 0.373550 | 0.444274 | 0.514999 | 0.566773 | 0.585723 |

----NORMALIZED STRESS RESULTANT NRTHETA•(DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | 0.000000 | 0.052359 | 0.090689 | 0.104718 | 0.090689 | 0.052359 | 0.000000 |
| 0.1500 | 0.000000 | 0.018912 | 0.032757 | 0.037025 | 0.032757 | 0.018912 | 0.000002 |
| 0.2000 | -0.000000 | -0.002564 | -0.004442 | -0.005129 | -0.004442 | -0.002564 | -0.000000 |
| 0.2500 | -0.000000 | -0.019423 | -0.033641 | -0.038846 | -0.033641 | -0.019423 | -0.000000 |
| 0.3000 | -0.000000 | -0.034097 | -0.059258 | -0.068194 | -0.059058 | -0.034297 | -0.000000 |

TABLE IV (Continued)

THE DISTORTION OF THE SHELL GIVEN BELOW IS MEASURED RELATIVE TO THAT OF THE FACE-UP POSITION

----NORMALIZED DISPLACEMENT U TILDE•(DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | -0. | -0. | -0. | -0. | 0. | 0. | 0. |
| 0.1500 | -0.008222 | -0.007086 | -0.003981 | 0.000262 | 0.004501 | 0.007606 | 0.008742 |
| 0.2000 | -0.009794 | -0.008435 | -0.004724 | 0.000346 | 0.005415 | 0.009127 | 0.010485 |
| 0.2500 | -0.006722 | -0.005786 | -0.003228 | 0.000265 | 0.003759 | 0.006317 | 0.007253 |
| 0.3000 | -0. | -0. | -0. | -0. | 0. | 0. | 0. |

----NORMALIZED DISPLACEMENT V TILDE•(DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | -0.000000 | -0.000000 | -0.000002 | -0.000000 | -0.000000 | -0.000000 | -0.000000 |
| 0.1500 | 0.000000 | 0.004406 | 0.007631 | 0.000011 | 0.007631 | 0.004406 | 0.000000 |
| 0.2000 | 0.000000 | 0.005352 | 0.009270 | 0.010704 | 0.009270 | 0.005352 | 0.000000 |
| 0.2500 | 0.000002 | 0.003764 | 0.006519 | 0.007528 | 0.006519 | 0.003764 | 0.000000 |
| 0.3000 | -0.000000 | -0.000000 | -0.000000 | -0.000000 | -0.000000 | -0.000000 | -0.000000 |

----NORMALIZED DISPLACEMENT W TILDE•(DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | -0.064366 | -0.048871 | -0.026533 | 0.051302 | 0.109134 | 0.151478 | 0.166967 |
| 0.1500 | 0.231882 | 0.034448 | 0.041458 | 0.051033 | 0.060609 | 0.067619 | 0.074185 |
| 0.2000 | 0.397176 | 0.392937 | 0.073907 | 0.050643 | 0.027379 | 0.010349 | 0.024116 |
| 0.2500 | 0.149381 | 0.136003 | 0.299754 | 0.050127 | 0.000501 | -0.035828 | -0.049126 |
| 0.3000 | 0.194239 | 0.174845 | 0.121862 | 0.049481 | -0.022898 | -0.075083 | -0.095276 |

TABLE IV (Continued)

----NORMALIZED DISPLACEMENT UFLEX TILDE. (DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | -0.012328 | -0.027809 | 0.003418 | 0.018864 | 0.034309 | 0.045616 | 0.049755 |
| 0.1500 | -0.034367 | 0.000021 | 0.012007 | 0.028302 | 0.044756 | 0.056742 | 0.061130 |
| 0.2000 | 0.012259 | 0.015644 | 0.024892 | 0.037525 | 0.050157 | 0.059405 | 0.062790 |
| 0.2500 | 0.035658 | 0.037076 | 0.040951 | 0.046244 | 0.051538 | 0.055412 | 0.056031 |
| 0.3000 | 0.064636 | 0.063274 | 0.059555 | 0.054474 | 0.049393 | 0.045674 | 0.044313 |

----NORMALIZED DISPLACEMENT VFLEX TILDE. (DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | 0.000000 | 0.020730 | 0.035905 | 0.041460 | 0.035905 | 0.020730 | 0.000000 |
| 0.1500 | 0.000000 | 0.028390 | 0.049173 | 0.056780 | 0.049173 | 0.028390 | 0.000000 |
| 0.2000 | 0.000000 | 0.033892 | 0.058703 | 0.067784 | 0.058703 | 0.033892 | 0.000000 |
| 0.2500 | 0.000000 | 0.038162 | 0.066099 | 0.076324 | 0.066099 | 0.038162 | 0.000000 |
| 0.3000 | 0.000000 | 0.041557 | 0.071980 | 0.083115 | 0.071980 | 0.041557 | 0.000000 |

----NORMALIZED DISPLACEMENT WFLEX TILDE. (DIMENSIONLESS)----

| GAMMA/THETA(DEG.) | -90.00 | -60.00 | -30.00 | -0. | 30.00 | 60.00 | 90.00 |
|---|---|---|---|---|---|---|---|
| 0.1000 | 0.232015 | 0.233076 | 0.235975 | 0.239935 | 0.243895 | 0.246795 | 0.247856 |
| 0.1500 | 0.380954 | 0.361871 | 0.309733 | 0.238512 | 0.167292 | 0.115153 | 0.096069 |
| 0.2000 | 0.498492 | 0.463395 | 0.367514 | 0.236538 | 0.105561 | 0.009680 | -0.025415 |
| 0.2500 | 0.602553 | 0.553182 | 0.418298 | 0.234243 | 0.049788 | -0.085296 | -0.134466 |
| 0.3000 | 0.698942 | 0.636256 | 0.465201 | 0.231062 | -0.002877 | -0.174132 | -0.236016 |

## REFERENCES


1. J. W. Mar and F. Y. M. Wan, "Distortions and Stresses of Paraboloidal Surface Structures," Group Report 71G-1-1, Lincoln Laboratory, M.I.T. (9 January 1962), DDC 296298, H-476.
2. ______________, "Distortions and Stresses of Paraboloidal Surface Structures. Part II. The Membrane Behavior," Group Report 71G-1-II, Lincoln Laboratory, M.I.T. (4 January 1963), DDC 296491, H-477.
3. ______________, "Distortions and Stresses of Paraboloidal Surface Structures. Part III," Group Report 71G-1-III, Lincoln Laboratory, M.I.T. (15 August 1963), DDC 417081, H-535.
4. ______________, "Distortions and Stresses of Paraboloidal Surface Structures," Group Report 71G-1 Part IV, Lincoln Laboratory, M.I.T. (29 April 1964), DDC 600125, H-579.
5. F. Y. M. Wan, "Membrane and Bending Stresses in Shallow, Spherical Shells," Technical Report 317, Lincoln Laboratory, M.I.T. (8 November 1964).

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13. ABSTRACT

The primary design requirement of a high-performance antenna is that the reflecting surface remain paraboloidal. For an antenna housed in a radome, strength considerations play a minor design role. Therefore, the antenna must have adequate structural stiffness accompanied by minimum weight. The basic structural components of the antenna are paraboloidal panels, which, when joined together, form a surface of revolution. Such a structural configuration, if properly fabricated, can be considered as a shell. Shell structures derive many of their attractive features from their two-dimensional surface nature, which brings with it geometrical complications to the strain-deflection relations and the equilibrium equations. Although the deflections of trusses, beams, and space frameworks are well understood, well documented, and easily obtained, this is not true for shells. In fact, shell behavior is currently the major topic of study in the structural mechanics field. The available solutions for even simple loadings of simple shells are of such a form that numerical results are not easily obtained. For these reasons, Lincoln Laboratory has been actively studying the deformations of paraboloidal shells. This user's manual will describe the capability, potentiality, and idiosyncrasies of the various LLAPS computer programs which are products of the above study.

14. KEY WORDS

paraboloidal shells antenna design computer program